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Investigation of the Effect of Frictional Contact in III-Mode Crack under Action of the SH-Wave Harmonic Load

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Abstract: The frictional contact interaction of the edges of a finite plane crack is studied for the case of normal incidence of a harmonic SH-shear wave which produces antiplane deformation. The forces of contact interaction and displacement discontinuity are analyzed. Influence of the wave frequency on the stress intensity factor for different coefficients of friction is studied here.

Keyword: Crack, friction, wave, BIE, fracture.

1 Introduction

In numerous publications of Guz and Zozulya it was shown that in the cracked body under action of a harmonic loading, taking into account of the crack edge contact interaction is very important. The problem of the crack edges contact interaction in 2-D and 3-D elastodynamics have been investigated in details in monograph [Guz and Zozulya (1993)] published in Russian and reviews [Guz and Zozulya (1995), (2001) and (2004)] published in English. For more detailed information see bibliography cited there. In above mentioned publications boundary element method has been used. Alternatively meshless methods can be used. For resent development in this area see [Atluri (2004), Atluri S.N., Liu H.T., Han Z.D. (2006a), Atluri S.N., Liu H.T., Han Z.D. (2006b)]. In some situation under action of the harmonic loading antiplane deformation in vicinity of crack may occur. In the case if there is load perpendicular to the crack surface, the crack edges are in a close contact and their frictional interaction take place. Influence of the frictional contact interactions of the crack edges on fracture mechanics criterions in the III-mode cracks is not investigated yet. This lack is made up in this publication.

2 Statement of the problem

Let consider an unbounded homogeneous isotropic elastic body in 3-D Euclidean space. There is a finite crack located in the plane $R^2 = \{\mathbf{x} : x_3 = 0\}$. The crack surface is described by its Cartesian coordinates

$$\Omega = \{x : -l \leq x_1 \leq l, x_2 = 0, -\infty \leq x_3 \leq \infty\}$$

A harmonic horizontally polarised shear SH-wave with frequency ω propagates in the plane R^2. The shear axis and axis Ox_3 are coinciding as it is shown in Fig.1.

The incident wave is defined by the potential

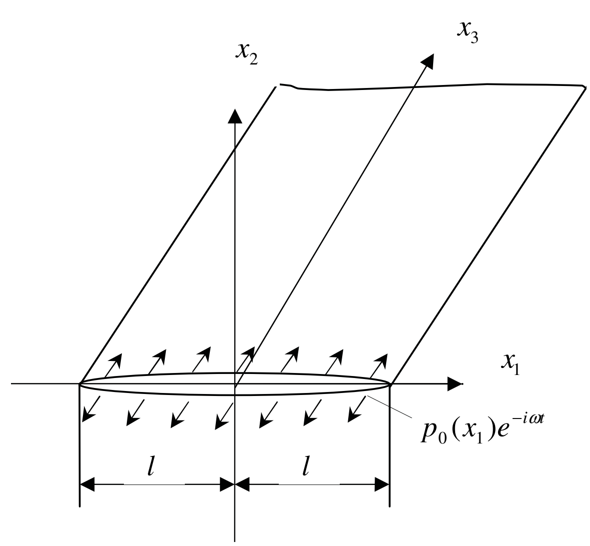

Figure 1: Finite crack under antiplane deformation

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function

$$\psi(\mathbf{x}_\alpha,t)=\psi_0 e^{i(k_2\mathbf{n}\cdot\mathbf{x}_\alpha-\omega t)},$$
$$k_2=\omega/c_2,\quad c_2=\sqrt{\mu/\rho}$$

where ψ_0 is the amplitude, k_2 is the wave number, c_2 is the velocity of the SH-wave, $\omega=2\pi/T$ is the frequency, T is the period of wave propagation, μ are the Lame constant, and ρ is the density of the material, $\mathbf{n}=(\cos\alpha,\sin\alpha)$ is the unit vector, normal to the front of the incidence wave, α is the angle of the incident wave, $\mathbf{x}_\alpha=(x_1,x_2)$ are Cartesian coordinates in the plane R^2.

This wave generate the stress-strain state that depend on two space coordinates $\mathbf{x}_\alpha\subset R^2$ and time $t\in\Im$ and is called antiplane deformation [Achenbach (1973)]. The deformation is described by the shear component $u_3(\mathbf{x}_\alpha,t)$. The corresponding stress tensor components are defined by Hooke's law

$$\sigma_{3\alpha}=\mu\partial_\alpha u_3. \tag{1}$$

The stress equation of motion has the form

$$\mu\partial_\beta\sigma_{3\beta}+b_3=\rho\partial_t^2u_3 \tag{2}$$

Here ∂_β and ∂_t are derivatives with respect to the coordinates and time, respectively, b_3 is the volume force.

Eliminating the $\sigma_{3\alpha}$ from Eq. 2 using Eq. 1 we find that the displacement $u_3(\mathbf{x}_\alpha,t)$ is governed by the scalar wave equation of the form

$$\mu\partial_\beta\partial_\beta u_3+b_3=\rho\partial_t^2u_3,\quad\forall(\mathbf{x}_\alpha,t)\in V\times\Im \tag{3}$$

Wave propagation in cracked body is a classical diffraction problem [Achenbach (1973), Guz and Zozulya (1993)]. Usually this problem may be divided in two separate problems: the problem for incident waves and the problem for reflection waves. Obviously, the problem for incident wave is trivial in the case under consideration. If the wave function $\psi(\mathbf{x}_\alpha,t)$ is known, then the components of the stress tensor and displacements vector under action of the incident wave are determined in the form

$$u_3=-\partial_2\psi=-ik_2n_2\psi_0e^{i(k_2\mathbf{n}\cdot\mathbf{x}_\alpha-\omega t)},$$
$$\sigma_{3\alpha}=\mu\partial_\alpha\partial_2\psi=\mu k_2^2n_\alpha n_2\psi_0e^{i(k_2\mathbf{n}\cdot\mathbf{x}_\alpha-\omega t)}$$

Therefore we will pay more attention to solution of the problem for reflected waves.

The on the crack's edges $n_1=0$ and $x_2=0$, therefore load caused by incident wave has the form

$$p_3(x_1,t)=-\sigma_{32}(x_1,t)=p_0e^{i(k_2x_1-\omega t)},$$
$$p_0=-\mu k_2^2\psi_0$$

With considering of the crack edges contact interaction the load on the crack edges has the form

$$p_3^s(x_1,t)=\begin{cases}p_3(x_1,t), & \forall x_1\notin\Omega_e\\ p_3(x_1,t)+q_3(\mathbf{x}_\alpha,t), & \forall x_1\in\Omega_e\end{cases}$$

where $\Omega_e=\Omega^+\cap\Omega^-$ is a region of close frictional contact, which is varied during time.

The force of the crack edges contact interaction q_3 and displacement discontinuity $\Delta u_3=u_3^+-u_3^-$ should satisfy the contact constrains in form of Coulomb friction [Panagiotopoulos (1985)]

$$\begin{aligned}&|q_3|\le k_*q_n\to\partial_t\Delta u_3=0,\\&|q_3|=k_*q_n\to\partial_t\Delta u_3=-\lambda_*q_3,\end{aligned} \tag{4}$$

where k_* and λ_* are coefficients dependent on the contacting surfaces properties, q_n is the normal to the crack surface force of contact interaction. In the problem under consideration we assume that it is known beforehand.

Because of contact constrains defined by Eq. 4 the problem under consideration is nonlinear. Therefore, as it was shown in [Guz and Zozulya (1993), (1995), (2001) and (2004)], the problem for reflected waves presents the periodic steady-state, but not harmonic, process. As a result, components of the stress-strain state, caused by the reflected waves can not be represented as functions of coordinates $\mathbf{x}_\alpha$, multiplied by factor $e^{-i\omega t}$, as it is usually does in elastodynamics in the case of the harmonic loading [Achenbach (1973)]. That is why we have to expand components of the displacement vector and stress tensor into Fourier series with the parameter of loading ω

$$\begin{aligned}u_3(\mathbf{x}_\alpha,t)&=\mathrm{Re}\left\{\sum_{-\infty}^{\infty}u_3^k(\mathbf{x}_\alpha)e^{i\omega_kt}\right\},\\ \sigma_{3\beta}(\mathbf{x}_\alpha,t)&=\mathrm{Re}\left\{\sum_{-\infty}^{\infty}\sigma_{3\beta}^k(\mathbf{x}_\alpha)e^{i\omega_kt}\right\}\end{aligned} \tag{5}$$

where

$$u_3^k(\mathbf{x}_\alpha) = \frac{\omega}{2\pi}\int_0^T u_3(\mathbf{x}_\alpha, t)e^{i\omega_k t}dt,$$

$$\sigma_{3\beta}^k(\mathbf{x}_\alpha) = \frac{\omega}{2\pi}\int_0^T \sigma_{3\beta}(\mathbf{x}_\alpha, t)e^{i\omega_k t}dt$$

In the same way, traction $p_3(x_1,t)$ on the crack edges and their opening $\Delta u_3(x_1,t)$ may be expanded into Fourier series

$$p_3(x_1,t) = \mathrm{Re}\left\{\sum_{-\infty}^{\infty} p_3^k(x_1)e^{i\omega_k t}\right\},$$

$$\Delta u_3(x_1,t) = \mathrm{Re}\left\{\sum_{-\infty}^{\infty} \Delta u_3^k(x_1)e^{i\omega_k t}\right\}$$

where

$$p_3^k(x_1) = \frac{\omega}{2\pi}\int_0^T p_3(x_1,t)e^{i\omega_k t}dt,$$

$$\Delta u_3^k(x_1) = \frac{\omega}{2\pi}\int_0^T \Delta u_3(x_1,t)e^{i\omega_k t}dt$$

3 Integral equations and fundamental solutions

Inserting Fourier series expansions defined by Eq. 5 into governing equation, instead of the wave equation defined by Eq. 3 we obtain a countable set of steady-state wave equations of the from

$$\partial_\beta\partial_\beta u_3^k + \frac{\omega^2}{c_2^2}u_3^k + \frac{\rho}{\mu}b_3^k = 0, \;\; k = 0, \pm 1, \pm 2, \ldots, \infty \tag{6}$$

In [Guz and Zozulya (1993), (1995), (2001) and (2004)] it was shown that the Fourier series expansions of the displacement discontinuity $\Delta u_3^k(\mathbf{x}_\alpha)$ and traction $p_3^k(\mathbf{x}_\alpha)$are related by the boundary integral equations (BIE) of the form

$$p_3^k(\mathbf{x}_\alpha) = -F.P.\int_\Omega F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)\Delta u_3^k(\mathbf{y}_\alpha)d\Omega,$$

$$k = 0, \pm 1, \pm 2, \ldots, \pm\infty, \quad \forall \mathbf{x}_\alpha \in \Omega. \tag{7}$$

3.1 Fundamental solutions

The kernels $F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ may be obtained from the fundamental solutions $U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ for the steady-state wave equations defined by Eq. 6. The fundamental solutions for the steady-state wave equation is well known and may be find anywhere, for example in [Dominguez (1993)]. It has the form

$$U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) = \frac{i}{4\mu}H_0^{(1)}\left(l_2^k\right), \quad l_2^k = r\omega_k/c_2$$

where $r = |\mathbf{x}_\alpha - \mathbf{y}_\alpha| \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$ is the length between points $\mathbf{x}_\alpha$ and $\mathbf{y}_\alpha$, $H_0^1(z)$ is the Bessel function of the third kind and zero order (Hankel function) [Abramowitz and Stegun (1964)].

Applying the operator of derivation in normal direction twice with respect to x_1 and y_2 correspondingly to the fundamental solutions $U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ we obtain

$$F_3(\mathbf{x}_\alpha, \mathbf{y}_\alpha, \omega_k) = -\mu^2\partial_n\partial_n U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k),$$

where $\partial_n = n_\alpha\partial_\alpha$ is the normal derivative.

The normal derivatives of the fundamental solutions $U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ are calculated in the form

$$\partial_n U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) = \frac{n_\alpha x_\alpha}{r}\frac{dU_3(r,\omega_k)}{dr}$$

$$\partial_n\partial_n U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha) = \frac{n_\alpha n_\beta}{r}\left(\delta_{\alpha\beta} - \frac{x_\alpha x_\beta}{r^2}\right)\frac{dU_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha)}{dr} + \frac{n_\alpha x_\alpha}{r}\frac{n_\beta x_\beta}{r}\frac{d^2U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha)}{dr^2}$$

In these equations we have used expressions for normal derivatives of the function r which is the distance between points $\mathbf{x}_\alpha$ and $\mathbf{y}_\alpha$ in the form

$$\partial_n r = \frac{n_\alpha x_\alpha}{r}, \quad \partial_n(\partial_n r) = \frac{n_\alpha n_\beta}{r}\left(\delta_{\alpha\beta} - \frac{x_\alpha x_\beta}{r^2}\right)$$

Taking into account that in the case under consideration $n_1 = 0$, $n_2 = 1$ and $x_2 = 0$, the normal derivative for the fundamental solution $U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ have the form

$$\partial_n\partial_n U_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) = \frac{dU_3(r,\omega_k)}{rdr}.$$

Using formula for the derivative of the Bessel function $H_0^1(z)$

$$\frac{d}{dr}H_0^{(1)}\left(\frac{\omega r}{c_2}\right) = -\frac{\omega}{c_2}H_1^{(1)}\left(\frac{\omega r}{c_2}\right)$$

we obtain the kernels $F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ for our specific case have the form

$$F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) = \mu\frac{i\omega_k}{rc_2}H_1^{(1)}\left(l_1^k\right)$$

These kernels are complex value functions. They may be represented in the form

$$F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) = F_3^{Re}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) + iF_3^{Im}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) \quad (8)$$

In order to separate real and imaginary parts of the fundamental solutions we substitute the Hankel functions by the Bessel functions of the first and the second kind using relation

$$H_\nu^{(1)} = J_\nu(z) + iY_\nu(z).$$

Now real and imaginary parts of the fundamental solutions have the form

$$\begin{aligned} F_3^{Re}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) &= -\mu\frac{\omega_k}{rc_2}Y_1(l_2) \\ F_3^{Im}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) &= \mu\frac{\omega_k}{rc_2}J_1(l_2) \end{aligned} \quad (9)$$

Let us consider in details the structure of the kernels $F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$. The Bessel functions of the first and second kind and the first order may be represented by the series expansion [Abramowitz and Stegun (1964)]

$$J_1(z) = \frac{z}{2}\left[1 - \frac{1}{1!2!}\left(\frac{z}{2}\right)^2 + \frac{1}{2!3!}\left(\frac{z}{2}\right)^4 - \dots\right]$$

$$\begin{aligned} Y_1(z) &= \frac{2}{\pi}(\ln(z/2) + \gamma)J_1(z) \\ &+ \frac{z}{2\pi}\left[1 - (1 + 1/2)\frac{1}{1!2!}\left(\frac{z}{2}\right)^2\right. \\ &\quad + (2 + 1 + 1/3)\frac{1}{2!3!}\left(\frac{z}{2}\right)^4 \\ &\quad \left. - (2 + 1 + 2/3 + 1/4)\frac{1}{3!4!}\left(\frac{z}{2}\right)^6 \dots\right] - \frac{2}{\pi z}. \end{aligned}$$

From these representations follows that

$$J_1(z) \to z \text{ and } Y_1(z) \to \frac{1}{z} \text{ for } z \to 0$$

and therefore for $\mathbf{x}_\alpha \to \mathbf{y}_\alpha$

$$F_3^{Re}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) \to \frac{1}{r^2}, \quad F_3^{Im}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k) \to 0$$

The kernels of these integral equations defined by Eq. 7 are hypersingular and must be considered in the sense of the Hadamard finite part in the same way as it was done in [Zozulya (2006a,b), Zozulya and Gonzalez-Chi (1999)] for 2-D and 3-D elastodynamic problems.

3.2 Boundary integral equations

From Eq. 8 it is follows that the BIE defined by Eq. 10 establish connection between complex functions $p_3^k(\mathbf{x}_\alpha)$ and $\Delta u_3^k(\mathbf{x}_\alpha)$. Separating real and imaginary parts

$$\begin{aligned} p_3^k(\mathbf{x}_\alpha) &= p_3^{Re}(\mathbf{x}_\alpha, k) + ip_3^{Im}(\mathbf{x}_\alpha, k), \\ \Delta u_3^k(\mathbf{x}_\alpha) &= \Delta u_3^{Re}(\mathbf{x}_\alpha, k) + i\Delta u_3^{Im}(\mathbf{x}_\alpha, k) \end{aligned}$$

and using representation from Eq. 8 for the kernels $F_3(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)$ we obtain the system of integral equations for each k in the form

$$\begin{aligned} & p_3^{Re}(\mathbf{x}_\alpha, k) = \\ & F.P.\int_\Omega F_3^{Re}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)\Delta u_3^{Re}(\mathbf{y}_\alpha, k)d\Omega \\ & -\int_\Omega F_3^{Im}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)\Delta u_3^{Im}(\mathbf{y}_\alpha, k)d\Omega \\ & p_3^{Im}(\mathbf{x}_\alpha, k) = \\ & \int_\Omega F_3^{Im}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)\Delta u_3^{Re}(\mathbf{y}_\alpha, k)d\Omega \\ & + F.P.\int_\Omega F_3^{Re}(\mathbf{x}_\alpha - \mathbf{y}_\alpha, \omega_k)\Delta u_3^{Im}(\mathbf{y}_\alpha, k)d\Omega \end{aligned} \quad (10)$$

The BIEs are solved numerically, transforming them into linear finite-dimensional system of the boundary element equations using collocation method. In [Menshykov, Menshykova M. and Wendland, (2005)] it was shown that solutions of

2-D elastodynamic contact problem obtained using collocation and Galerkin BIE methods coincides for all considered frequencies. For zero order interpolation polynomials with piecewise constant approximation of the boundary elements the system of boundary element equations has the form

$$\begin{aligned} p_3^{Re}(x^m,\omega_k) &= \sum_{n=1}^{N}\Big[F_3^{Re}(x^m-y^n,\omega_k)\Delta u_3^{Re}(y^n,k) \\ &\quad - F_k^{Im}(x^m-y^n,\omega_k)\Delta u_3^{Im}(y^n,k)\Big] \\ p_3^{Im}(x^m,k) &= \sum_{n=1}^{N}\Big[F_3^{Im}(x^m-y^n,\omega_k)\Delta u_3^{Re}(y^n,k) \\ &\quad - F_3^{Re}(x^m-y^n,\omega_k)\Delta u_3^{Im}(y^n,k)\Big] \end{aligned} \tag{11}$$

Their coefficients are defined by the equations

$$F_k^{Re}(x^m-y^n,\Delta_n) = \int\limits_{2\Delta_n} F_3^{Re}(x^m-y,\omega_k)dy \tag{12}$$

$$F_k^{Im}(x^m-y^n,\Delta_n) = \int\limits_{2\Delta_n} F_3^{Im}(x^m-y,\omega_k)dy \tag{13}$$

Let us consider in detail how to calculate these coefficients. If points $\mathbf{x}_\alpha$ and $\mathbf{y}_\alpha$, belong to different boundary elements the integrals in Eq. (12) and Eq. (13) are non-singular and their calculation gives no difficulties. For example, the Gauss quadrature formula may be applied to the numerical calculation of those integrals. If points $\mathbf{x}_\alpha$ and $\mathbf{y}_\alpha$, belong to the same boundary elements the integrals in Eq. 12 are hypersingular. For their calculation the finite parts of the divergent integrals according to Hadamard will be used. We will use for this purpose the relationship

$$\begin{aligned} &F_k^{Re}(x^m-y^n,\Delta_n) = \\ &\int\limits_{2\Delta_n}\Big[F_3^{Re}(x^m-y,k)-F_3^{St}(x^m-y)\Big]dy \\ &\qquad\qquad + F.P.\int\limits_{2\Delta_n} F_3^{St}(x^m-y)dy \end{aligned}$$

Here $F_3^{St}(\mathbf{x}_\alpha-\mathbf{y}_\alpha) = \frac{2\mu}{\pi r^2}$ is the fundamental solution for the elastostatic problem, extracted from $F_3^{Re}(\mathbf{x}_\alpha-\mathbf{y}_\alpha,k)$ by a limit transition $\omega_k\to 0$. This fundamental solution is hypersingular. Calculating the finite part integral according to Hadamard we obtain

$$F_3^{St}(x^m-y^n,\Delta_n) = -\frac{4\mu}{\pi}\frac{\Delta_n}{(x^m-y^n)^2-\Delta_n^2}$$

Now the residuary integrals in Eq. 12 are regular and can be calculated using standard numerical approaches. Since the rectilinear crack under consideration, these regular integrals can be calculated analytically using metodology presented in [Guz and Zozulya (2001)]. For this purpose let us consider the integrals

$$\begin{aligned} \gamma_k^{mn}(\Delta_n,c_\alpha) &= \int\limits_{-\Delta_n}^{\Delta_n}(r^*\omega/2c_\alpha)^{2k}d\beta \\ &= \frac{(\omega/2c_\alpha)^{2k}}{2k+1}(r_{+\Delta}^{2k+1}+r_{-\Delta}^{2k+1}) \end{aligned}$$

$$\begin{aligned} &\int\limits_{-\Delta_n}^{\Delta_n}(r^*\omega/2c_\alpha)^{2k}\ln(r^*\omega/2c_\alpha)d\beta \\ &\qquad = \eta_k^{mn}(\Delta_n,c_\alpha)-\gamma_k^{mn}(\Delta_n,c_\alpha)/(2k+1) \end{aligned}$$

$$\begin{aligned} \eta_k^{mn}(\Delta_n,c_\alpha) = \frac{(\omega/2c_\alpha)^{2k}}{2k+1}\Big(& r_{+\Delta}^{2k+1}\ln(r_{+\Delta}^{\omega}/2c_\alpha) \\ &+ r_{-\Delta}^{2k+1}\ln(r_{-\Delta}^{\omega}/2c_\alpha)\Big) \end{aligned}$$

where $r^* = |y^m-x^n+\beta|$, $r_{+\Delta} = |y^m-x^n+\Delta_n|$, $r_{-\Delta} = |y^m-x^n-\Delta_n|$.

After integration of the Bessel functions we obtain the following expressions

$$\begin{aligned} J_1^*(m,n,c_\alpha) &= \int\limits_{-\Delta_n}^{\Delta_n}(\omega/c_\alpha r)J_1(l_\alpha^*)d\beta \\ &= (\omega/2c_\alpha)^2\left[2\Delta_n+\sum_{k=0}^{\infty}(-1)^k\frac{\gamma_k^{mn}(\Delta_n,c_\alpha)}{(k!)(k+1)!}\right] \end{aligned}$$

$$\begin{aligned} Y_1^*(m,n,c_\alpha) &= \int\limits_{-\Delta_n}^{\Delta_n}(\omega/c_\alpha r)Y_1(l_\alpha^*)d\beta \\ &= \gamma J_1^*(m,n,c_\alpha)-\left(\frac{\omega}{2c_\alpha}\right)^2(2\Delta_n+A)+2\left(\frac{\omega}{2c_\alpha}\right)^2 B \end{aligned}$$

where

$$A=\sum_{k=0}^{\infty}(-1)^{k}\frac{\gamma_{k}^{mn}(\Delta_{n},c_{\alpha})}{k!(k+1)!}(\psi(k+1)+\psi(k+2))$$

$$B=\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!(k+1)!}\frac{\eta_{k}^{mn}(\Delta_{n},c_{\alpha})-\gamma_{k}^{mn}(\Delta_{n},c_{\alpha})}{(2k+1)}$$

Utilization of these equations allows significantly reduce time of the boundary element equations coefficients calculation. In contrast with other approaches (see for example [Loeber and Sih (1968)]) the methods developed here demonstrate good accuracy where ω_k is low or high. The system of boundary element equations does not lose stability with increasing ω_k.

4 Algorithm for the problem solution.

For solution of the BIE defined by Eq. 11 with considering the unilateral constrains with friction defined by Eq. 4 we use the algorithms developed by Zozulya and presented in [Guz and Zozulya (2001), (2002), Zozulya and Menshykov O.V. (2003)]. The algorithm consists of the following steps:

- the initial distribution of the contact forces $q_3^0(x_1,t)$, $\forall \mathbf{x}\in\Omega$, $\forall t\in\Im$ is assigned;
- the problem without constrains is solved and the unknowns quantities on the contact surfaces $\Delta u_3(x_1,t)$ are defined;
- the normal and tangential components of the vector of contact forces are corrected to satisfy the unilateral restrictions

  $$q_3^{k+1}(x_1,t)=P_\tau\left[q_3^k(x_1,t)-\rho_\tau\partial_t\Delta u_3^{k+1}(x_1,t)\right],$$

  where,

  $$P_\tau[q_3]=\begin{cases}q_3(x_3,t), & \text{if } |q_3|\le k_\tau q_n(x_3,t)\\ k_\tau q_n(x_3,t)\frac{q_3}{|q_3|}, & \text{if } |q_3|>k_\tau q_n(x_3,t)\end{cases}$$

  is operator of the orthogonal projection onto set $|q_3|\le k_\tau q_n(x_3,t)$, coefficient ρ_τ has been chosen based on the conditions that give the best convergence of the algorithm;
- proceed to the next step of the iteration.

5 Numerical results

The case of normal incidence of a harmonic shear SH-wave which produce antiplane deformation is studied here. For simplicity we suppose that the force normal to the crack edges is constant along the crack length for any time and unit i.e. $q_n=1$. Assume that the incident shear SH-wave has the unit amplitude and the crack is located in the material with following mechanical characteristics: elastic modulus $E=200\,GPa$, Poisson's ratio $\nu=0.25$, specific density $\rho=7800\,kg/m^3$.

The crack opening and frictional contact force in the middle point of the crack ($x_1=0$) during period of the wave action for different wave numbers and coefficients of friction are presented in Fig. 2-7.

Graphs in Fig. 8-13 illustrate the crack opening and frictional contact forces distribution along the crack length during period of the wave action.

In our previous publications was shown that contact interaction affects fraction mechanics criterions. Let us study influence of the frictional contact interaction of the opposite crack surfaces on the stress intensity factor. For the tearing (or antiplane) mode III the displacement at the crack tip has the form

$$u_3=\frac{K_{III}}{\mu}\sqrt{\frac{2\varepsilon}{\pi}}\sin(\theta/2)$$

Using limit transition for $\theta=\pi$ we obtain

$$K_{III}=\lim_{\varepsilon\to 0}\frac{\mu\sqrt{\pi}}{\sqrt{2\varepsilon}}u_3(l-\varepsilon,t)$$

The stress intensity factor against wave number for different coefficients of friction is presented in Fig.14. The curves on the graphs correspond to: 1- solution without contact and 2 and 3 - with contact for $k_\tau=0.2$ and $k_\tau=0.4$ correspondently. We have to mention that solution obtained without crack edges contact interaction (curve 1) coincides with the one presented in [Loeber and Sih (1968)].

From these graphs follow that contact interaction of the crack edges has an influence on the crack opening and affects fracture mechanics criterions. It has to be taken into account in design constructions using methods of fracture mechanics.

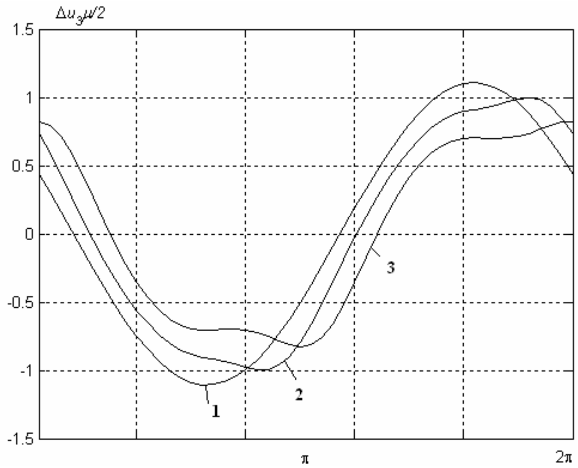

Figure 2: Crack opening $k_2 = 0.25$: $1 - k_\tau = 0$, $2 - k_\tau = 0.2$, $3 - k_\tau = 0.4$.

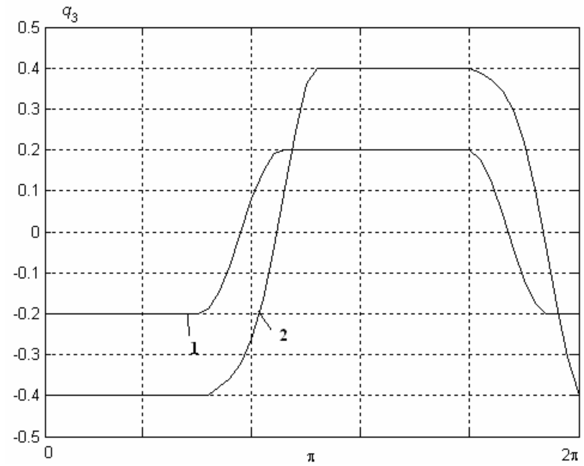

Figure 3: Friction contact force for $k_2 = 0.25$: $1 - k_\tau = 0.2$, $2 - k_\tau = 0.4$.

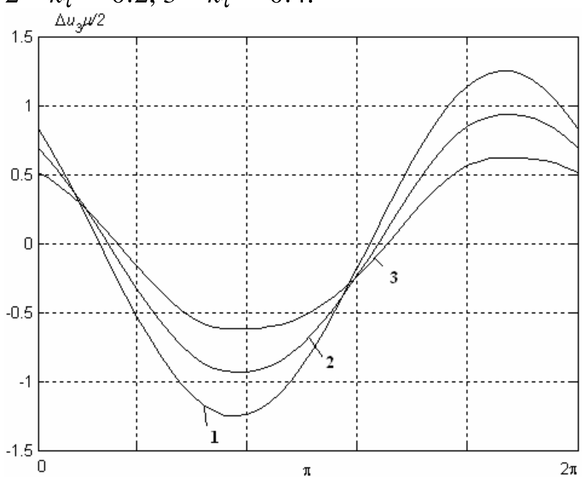

Figure 4: Crack opening $k_2 = 1.0$: $1 - k_\tau = 0$, $2 - k_\tau = 0.2$, $3 - k_\tau = 0.4$.

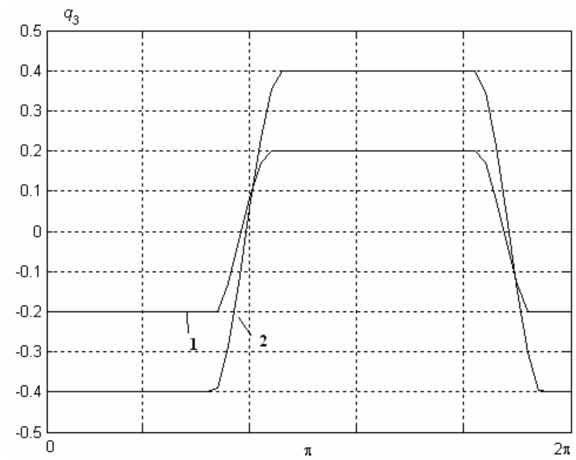

Figure 5: Friction contact force for $k_2 = 1.0$: $1 - k_\tau = 0.2$, $2 - k_\tau = 0.4$.

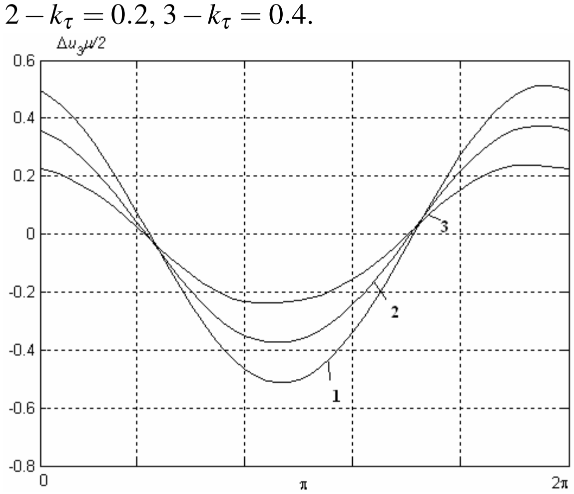

Figure 6: Crack opening $k_2 = 1.75$: $1 - k_\tau = 0$, $2 - k_\tau = 0.2$, $3 - k_\tau = 0.4$.

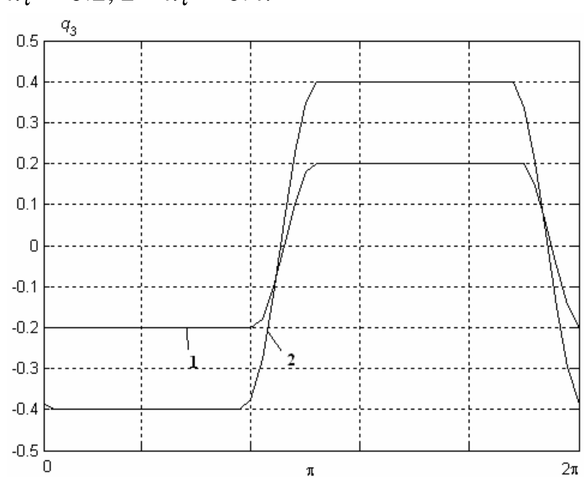

Figure 7: Friction contact force for $k_2 = 1.75$: $1 - k_\tau = 0.2$, $2 - k_\tau = 0.4$.

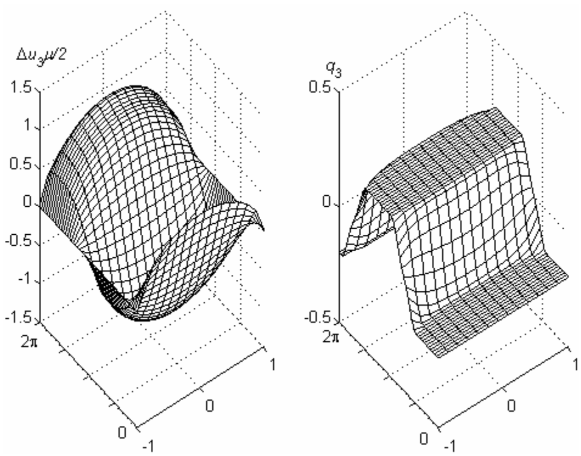

Figure 8: Crack opening and friction contact force for $k_2 = 0.25$ and $k_\tau = 0.2$.

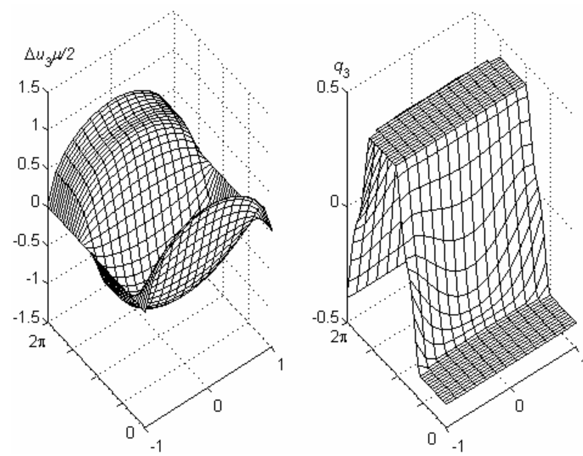

Figure 9: Crack opening and friction contact force for $k_2 = 0.25$ and $k_\tau = 0.4$.

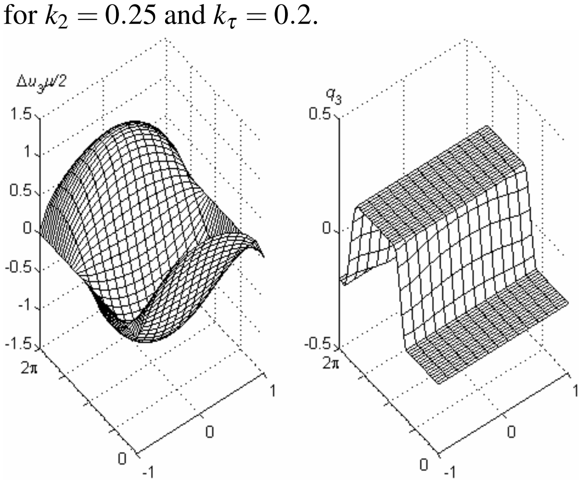

Figure 10: Crack opening and friction contact force for $k_2 = 1.0$ and $k_\tau = 0.2$.

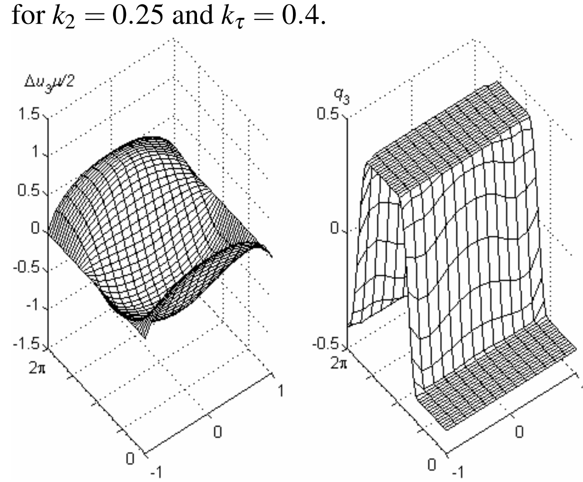

Figure 11: Crack opening and friction contact force for $k_2 = 1.0$ and $k_\tau = 0.4$.

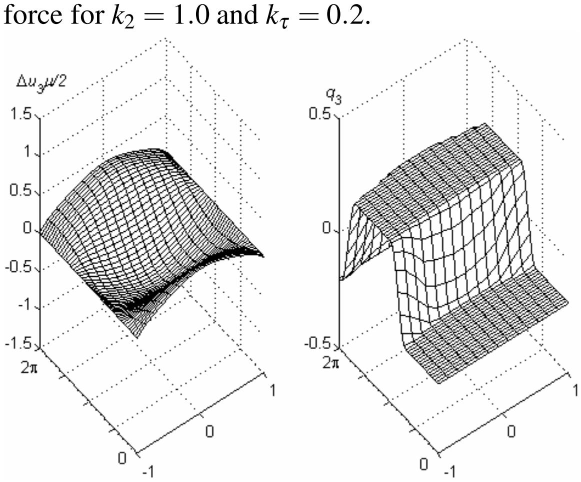

Figure 12: Crack opening and friction contact force for $k_2 = 1.75$ and $k_\tau = 0.2$.

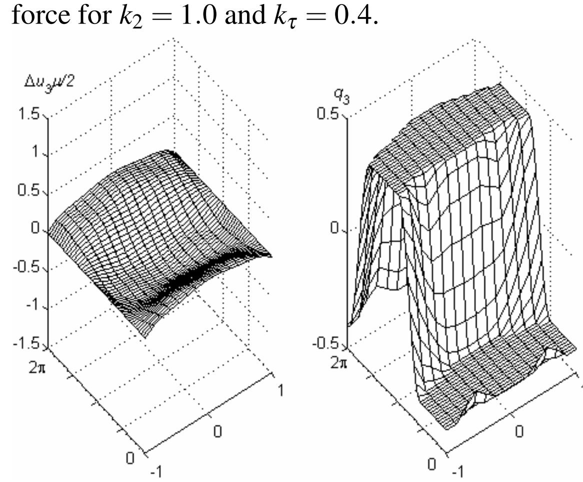

Figure 13: Crack opening and friction contact force for $k_2 = 1.0$ and $k_\tau = 0.4$.

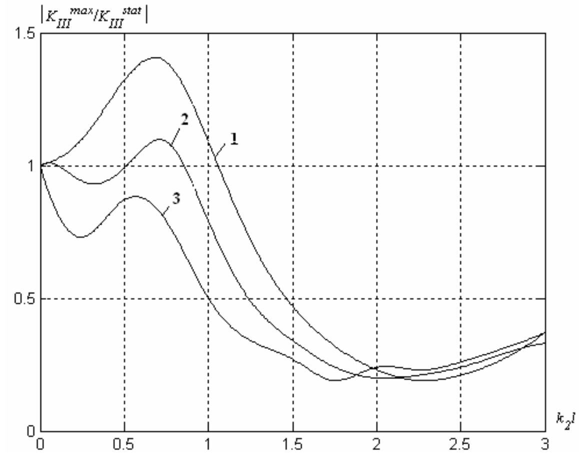

Figure 14: Stress intensity factor against wave number: $1-k_\tau=0$, $2-k_\tau=0.2$, $3-k_\tau=0.4$.

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