Copyright © 2007 Tech Science Press CMES, vol.17, no.3, pp.231-238, 2007

The Moving Finite Element Method Based on Delaunay Automatic triangulation For Fracture Path Prediction Simulations In Nonlinear Elastic-Plastic Materials

T. Nishioka[1], **Y. Kobayashi**[1] **and T. Fujimoto**[1]

Abstract: First, for growing cracks in elastic-plastic materials, an incremental variational principle is developed to satisfy the boundary conditions near newly created crack surfaces. Then using this variational principle, a moving finite element method is formulated and developed, based on the Delaunay automatic triangulation. Furthermore, theoretical backgrounds on numerical prediction for fracture path of curving crack using T* integral are explained. Using the automatic moving finite element method, fracture-path prediction simulations are successfully carried out.

Keyword: Nonlinear Fracture Parameter, T^* Integral, Moving Finite Element Method, Delaunay Triangulation.

1 Introduction

Numerical prediction of fracture path is an extremely important research subject not only for academic interest but also for the establishment of a safety design methodology that prevents catastrophic overall failures of structures. However, numerical fracture-path predictions of nonlinear fracture phenomena have not fully been succeeded, due to various inherent difficulties.

In previous studies, for fracture path predictions in elastic materials, we developed a moving finite element method based on Delaunay automatic triangulation. This made it possible to predict dynamic crack kinking and curving fracture paths under impact loading. Furthermore, the moving finite element method was extended by Nishioka, Tchouikov and Fujimoto (2001) to dynamic crack branching problems, and demonstrated excellent prediction of dynamic crack branching paths.

In this study, to establish a simulation method for complex crack propagation in nonlinear materials, first, we derive an incremental variational principle to satisfy the boundary conditions near newly created crack surfaces. Using this variational principle, we formulate a moving finite element equation. Then, based on this variational principle and Delaunay automatic triangulation technique proposed by Sloan and Houlsby (1984) and Taniguchi (1992), we develop a moving finite element method based on Delaunay automatic mesh generation.

In this study, the authors used the T* integral derived by Atluri, Nishioka and Nakagaki (1984) to numerical prediction for fracture path of a curving crack in a nonlinear material. Thus, the theoretical backgrounds on numerical prediction using the T* integral for fracture path of curving crack are presented. To demonstrate the applicability of the present methodology, we carried out the elastic-plastic fracture path prediction for a curving crack under a mixed-mode condition. The pertinent results of this numerical prediction are also presented.

The applicability of the T* integral has been studied by several researchers (Okada and Atluri(1999), Kobayashi and Atluri(1998), Okada and Atluri(1997), Kobayashi and Atluri(2001), Brust, Nishioka, Atluri and Nakagaki(1985), Nishioka(2005), Fujimoto and Nishioka(2005)).

2 Moving Finite Element Method Based On Delaunay Automatic Mesh Generation

To simulate crack propagation by finite element method two different concepts of computational modeling can be considered, i.e. (i) the station-

[1] Faculty of Maritime Sciences, Kobe University, Kobe, Japan

ary element procedure (or fixed element procedure), and (ii) the moving element procedure, as reviewed by Nishioka and Atluri(1986), and Nishioka (1994, 1997).

For elastic-plastic crack propagation problem, the fixed finite element procedure has the following severe disadvantages:

(1) The boundary conditions near propagating crack tip cannot be satisfied exactly.

(2) The fixed mesh pattern may not fit with actual fracture paths.

To overcome the aforementioned difficulties, Nishioka and coworkers have developed various types of moving finite element method.

Recently the concept of moving finite element method was extended by Nishioka, Tokudome and Kinoshita (2001) to dynamically curving and kinking fracture problems using the modified Delaunay automatic triangulation proposed by Taniguchi (1992).

In this study, we further extend the moving finite element method based on Delaunay automatic triangulation to elastic-plastic crack propagation problem.

Generation phase simulation of dynamic crack bifurcation phenomenon using moving finite element method based on Delaunay automatic triangulation was successfully achieved by Nishioka, Furutsuka, Tchouikov and Fujimoto(2002).

2.1 Modified Delaunay automatic triangulation

In the modified Delaunay triangulation, only exterior and interior boundary points and specified interior points (if they are necessary) are required for automatic mesh generation.

Let us consider to generating mesh for a cracked body whose initial crack opening is very small. Points on boundaries are placed at first. At this time, to distinguish the upper and lower crack surfaces (see Fig.1(a)), the coordinates of the upper and lower crack surfaces are shifted by infinitesimally small distances $\pm\varepsilon$ toward the perpendicular direction to the crack surfaces. Thus, the crack is opened by 2ε. Due to the stress singularity at the crack tip, the specified interior points are placed around the crack tip also (Fig.1 (b)). Then the mesh pattern is automatically generated using exterior boundary points and the specified interior points(Fig.1 (c)).

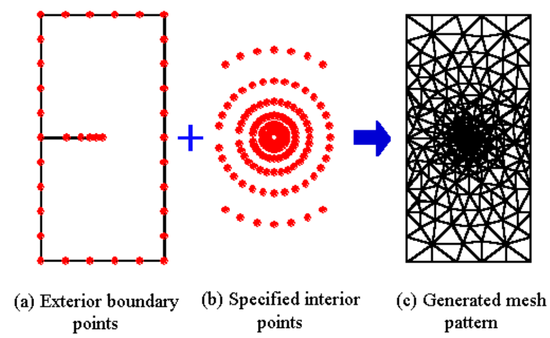
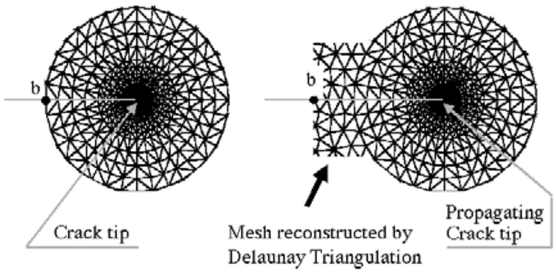


Figure 1: An example of mesh generation

2.2 Automatic mesh generation for propagating crack

Nishioka and coworkers have developed various types of moving finite element procedure. These are reviewed and summarized by Nishioka and Atluri (1986) and Nishioka(1994, 1997). In this study, the concept of the moving finite element method is extended to complex crack propagation problems using the modified Delaunay automatic triangulation.

In the moving finite element method based on Delaunay automatic triangulation, a crack advanced as shown Fig. 2. The group of the specified interior points around the propagating crack tip translates in each step for which crack growth occurs. In each step, the previous crack tip point breaks into two nodal points. The crack tip always remains at the center of the group of the moving elements throughout the analysis even for complicated crack propagation. At each step, the interior region between the specified nodes around the crack tip and specified boundary nodes, is automatically broken into triangular elements by using the modified Delaunay automatic triangulation.



(a) Previous mesh pattern (b) Present mesh pattern

Figure 2: Moving finite element procedures for a propagating crack

3 Variatinal Principle for a propagating crack in a Nonlinear Material

Let us consider a propagating crack in an elastic-plastic material. Here we use the incremental infinitesimal deformation theory. In the moving finite element procedure, the mesh pattern near the propagating crack tip translates in each time step as illustrated in Fig. 2. After the mesh translation, the moving finite element method requires the mapping of solution fields in the previous mesh onto those in the present mesh. To satisfy the governing equations and the boundary conditions in the present mesh at time t=$t_0+\Delta$t, a new incremental variational principle is derived as follows:

$$\begin{aligned}\int_V (\Delta\sigma_{ij}\delta\varepsilon_{ij}+\rho\Delta\ddot{u}_i\delta u_i)dV - \int_V \Delta\overline{f_i}\delta u_i dV \\ -\int_{S_t}\Delta\overline{t_i}\delta u_i dS \\ = +\int_v \overline{f_i^{t_0}}\delta u_i dV + \int_{\overline{t_i^{t_0}}}\delta u_i dS \\ -\int_{S_t}(\sigma_{ij}^{t_0}\delta\varepsilon_{ij}+\rho\ddot{u}_i^{t_0})dV \quad (1)\end{aligned}$$

Where the superscript t_0 denotes a quantity of the previous step ($t=t_0$) in the present mesh pattern at time t. The new crack surfaces are created by the crack propagation during time increment Δt. The finite element solution field obtained from the new variational principle satisfies the following equilibrium equation and the mechanical boundary condition including the crack surfaces at time $t=t_0+\Delta t$:

$$\begin{aligned}\Delta\sigma_{ij,j}+\Delta\overline{f_i}-\rho\Delta\ddot{u}_i &= -(\sigma_{ij,j}^{t_0}+\overline{f_i^{t_0}}-\rho\ddot{u}_i^{t_0}) \\ \Delta\sigma_{ij}n_j &= \overline{t_i^{t_0}}+\Delta\overline{t_i}-\sigma_{ij}^{t_0}n_j\end{aligned} \quad (2)$$

The finite element method was developed based on Eq. (1)

4 The T^* integral

The global components the T* integral T_k^* can be expressed as

$$\begin{aligned}T_k^* = &\int_{\Gamma+\Gamma_c}[(W+K)n_k - t_i u_{i,k}]dS \\ &+\int_{V_\Gamma - V_\varepsilon}\left[\rho\ddot{u}_i u_{i,k}-\rho\dot{u}_i\dot{u}_{i,k}+\sigma_{ij}\varepsilon_{ij,k}-W_{,k}\right]dV\end{aligned} \quad (3)$$

Where W and K are the stress working and kinetic energy densities, respectively. Integral paths are shown Fig. 3.

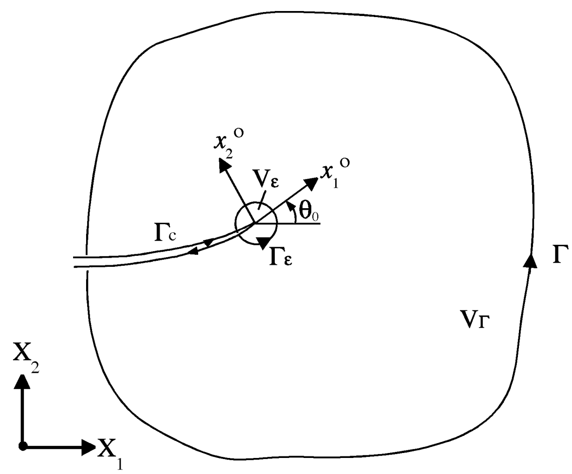


Figure 3: Integral Paths

Γ is a far-field contour that encloses the crack tip and envelops a volume V_Γ; Γ_ε is a near-field contour arbitrarily close to the propagating crack tip and envelops a small volume V_ε; and Γ_c is the crack surface enclosed by Γ. Physically, the near-tip region V_ε can be considered as the process zone in which micro-processes associated with fracture occur.

For elastic material, the T* integral reduces to the J' integral derive by Nishioka and Atluri(1983) for elasto-dynamic cases. Moreover the T* integral reduces to the J integral derived by Budiansky and Rice (1973) for elastostaric cases.

4.1 Physical Meaning of the Components of the T^* integral

The crack-axis components of the T* integral T^{*0}_l can be easily obtained by the coordinate transformation:

$$T^{*0}_l = \alpha_{lk}(\theta_0)T^*_k \tag{4}$$

Where θ_0 is the crack direction. The tangential crack-axis component T^{*0}_1 has the meaning of the energy flow rate to the process zone for a propagating crack in an elastic-plastic material under a steady-state condition. The vertical crack-axis component T^{*0}_2 can be considered as a measure of mixed-mode state of the crack tip as reviewed in the paper [Nishioka and Fujimoto (2000)].

4.2 Vertical Crack-Axis Components of the T^* Integral

For an elasto-dynamically propagating crack in homogeneous material, the crack-axis components of the T* integral cat be related with stress intensity factors as derived by Atluri, Nishioka and Nakagaki (1984). Thus in this case, we have

$$T^{*0}_2 = J'^0_2 = -\frac{A_{IV}(C)}{\mu}K_I K_{II} \tag{5}$$

where $A_{IV}(C)$ is function of the crack velocity C. For a static crack ($C = 0$), this function is $A_{IV}(0) = (\kappa+1)/4$.

Contrary to tangential crack-axis component of the T^* integral T^{*0}_1, the vertical crack-axis component to the crack direction, i.e., T^{*0}_2 is zero under pure mode I and II conditions as can be seen by Eq.(5). for a fixed value of K_I, The absolute value of T^{*0}_2 increases for increasing K_{II} value. Moreover, the sign of T^{*0}_2 can be used to judge the direction of inplane shearing mode, For instance, T^{*0}_2 is negative if K_{II} is positive, and vice versa. Thus the vertical crack-axis components of T^* can be used as a measure that expresses the magnitude of the mixed-mode state.

Similarly with the local symmetry criterion ($K_{II} = 0$ criterion), for curving crack propagation in a homogeneous material, $T^{*0}_2 = 0$ criterion can be postulated.

5 Mixed-Phase Simulation With Fracture-Path Prediction Mode

For non-self-similar fracture such as curving crack growth, three types of numerical simulation can be considered, as proposed by Nishioka (1997). First, the generation phase simulation can be conducted similarly with the generation phase simulation proposed by Kanninen (1978) for self-similar dynamic fracture, except additionally using experimental data on the curved fracture-path history. (see Fig.4 (i))

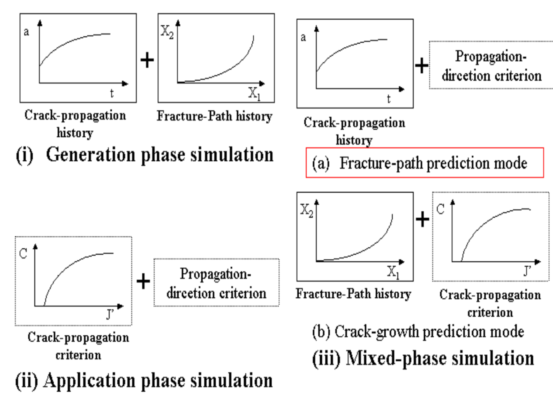


Figure 4: Types of fracture simulation

On the other hand, in the application phase simulation for curving crack growth, two criteria must be postulated or predetermined as shown Fig. 4 (ii). One is the crack-propagation criterion, and the other is the propagation-direction criterion. (see Fig.4 (ii))

To verify only the propagation-direction criterion, Nishioka (1997) has proposed "mixed-phase simulation" as depicted in Fig. 4 (iii).

In this study, to verify local symmetry criterion for curving crack propagation in a homogeneous elastic-plastic material ($T^{*0}_2 = 0$ criterion), we carried out the elastic-plastic fracture path prediction for a curving crack under mixed-mode condition using crack-propagation history obtained by experiments.

6 Experiments Before Fracture-Path Prediction Simulation

Kobayashi and coworkers (1999) carried out mixed mode elastic-plastic stable crack growth experiments.

Specimen geometry that they used is shown in Fig. 5, and its material is 2024-T3 Al alloy. Figure 6 shows constitutive relation for 2024-T3

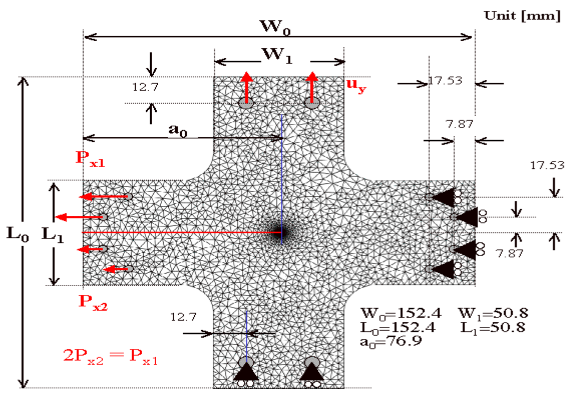


Figure 5: Specimen geometry

Al alloy. In this figure, experimental data providing by Kobayashi and coworkers (1999) are indicated by circle symbol. Based on these experimental data, the authors evaluate swift type constitutive equation as shown in this figure. Relation between displacement Uy and shear load

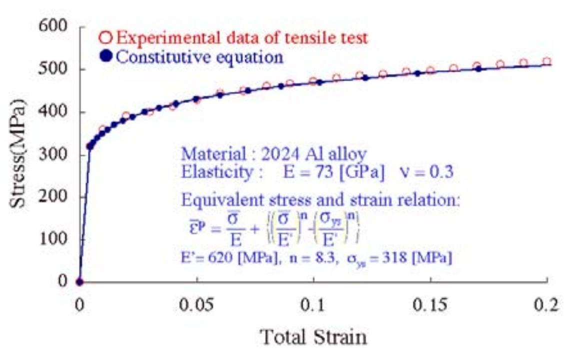


Figure 6: Constitutive Relation for 2024-T3 Al alloy

Px and relation between displacement Uy and crack propagation length Δa are shown in Fig. 7 and Fig. 8 obtained by experiments. Fitted curves of experimental data used in after-mentioned fracture-path prediction simulation are shown in these figures also.

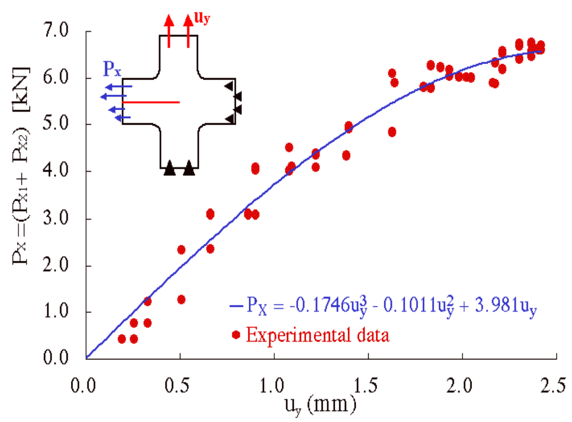


Figure 7: Relation between Displacement Uy and Shear Load Px

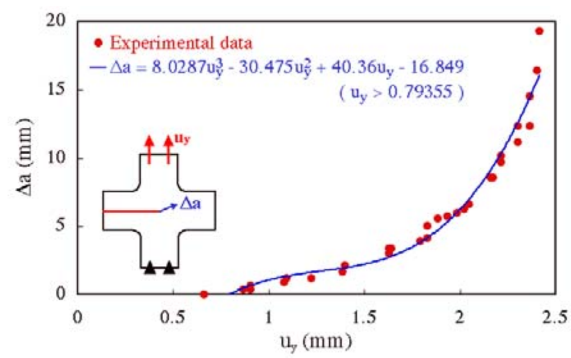
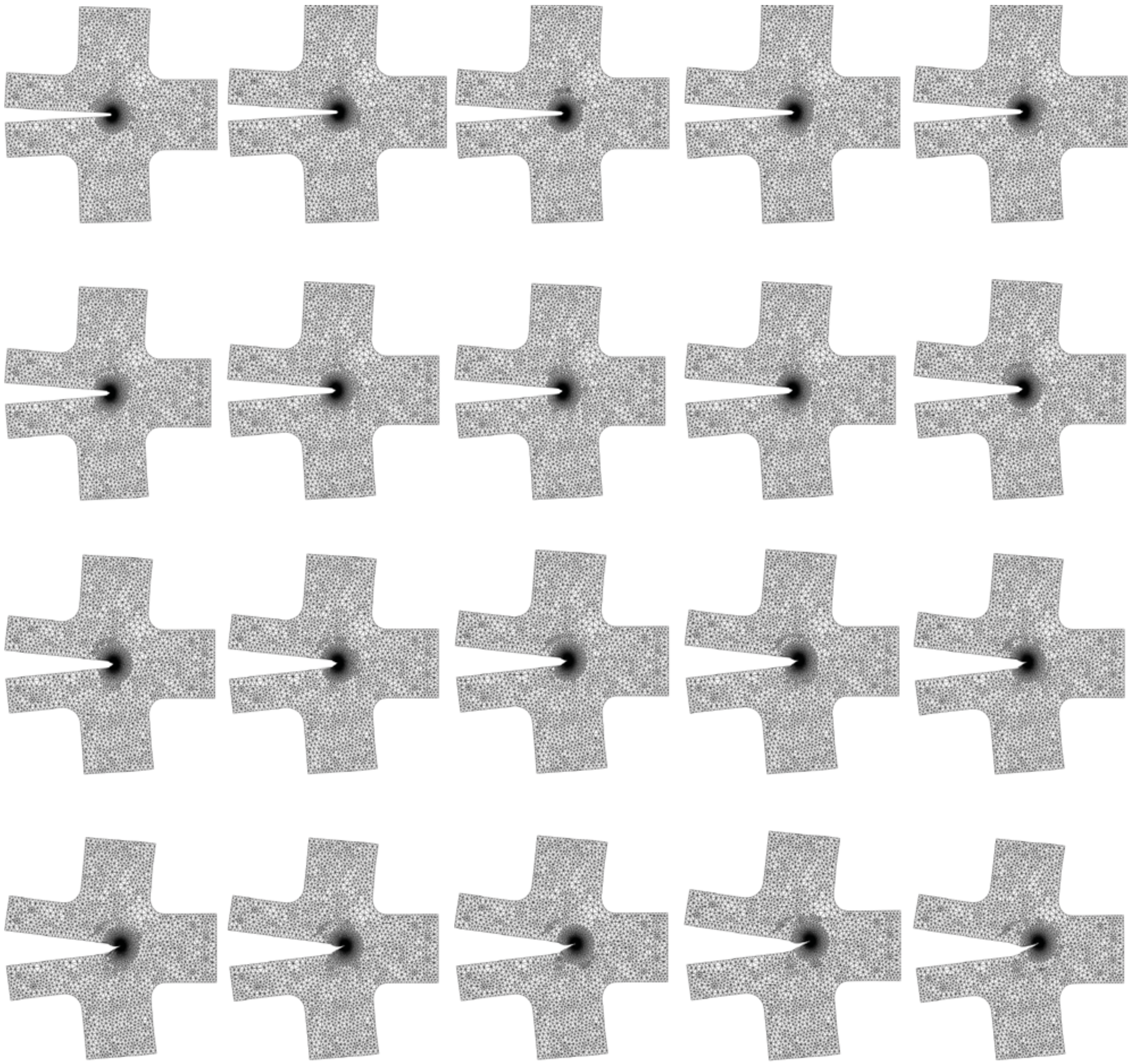


Figure 8: Relation between Displacement Uy and Crack Propagation Length Δa

7 Fracture-Path Prediction Simulation

In this study, we carried out fracture-path prediction phase simulations using afore-mentioned experimental relations and local symmetry criterion ($T^{*0}_2 = 0$) as criterion for crack propagation direction prediction.

The simulated fracture-paths using a moving finite element method and local symmetry criterion ($T^{*0}_2 = 0$) are shown in Fig. 9. In this figure, the magnification factor of displacement is ten.



$0.79355 \leqq u_v$ [mm], $3.00821 \leqq P_x$ [kN], $0.0 \leqq a \leqq 14.7$ [mm]

Figure 9: Mesh patterns on crack propagation progress

Figure 10 shows the distributions of equivalent plastic strain pattern. Large plastic deformations are seen around the crack-tip and the loading point.

T^* integrals are plotted in Fig. 11 against various integral paths. Excellent path independence can be seen even for elastic-plastic boundaries.

The history of T^* components are shown in Fig.12 It is seen that ${T^*}_2^0$ integral is almost zero during crack propagation. On the other hand, the ${T^*}_1^0$ integral is drastically reduces its value, and tends to reach nearly constant after a certain amount of crack extension.

In Fig. 13, predicted fracture path is compared with experimentally obtained fracture-paths. Since the reinforcing straps were placed on the far right side of the specimen, the numerical results can be compared with the reinforced specimens up to $X \sim 6$mm. Simulated fracture-path agrees well with experimental results.

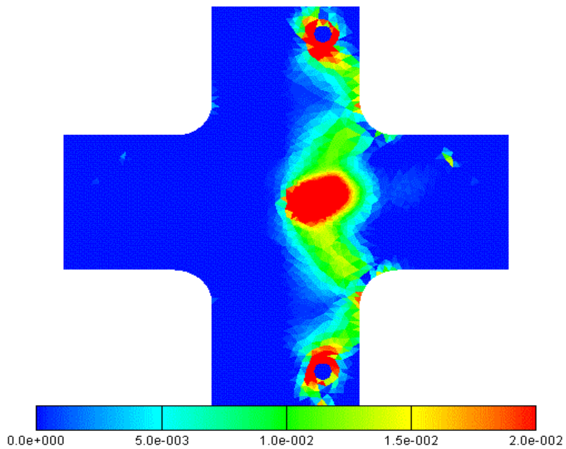


Figure 10: Equivalent plastic strain pattern

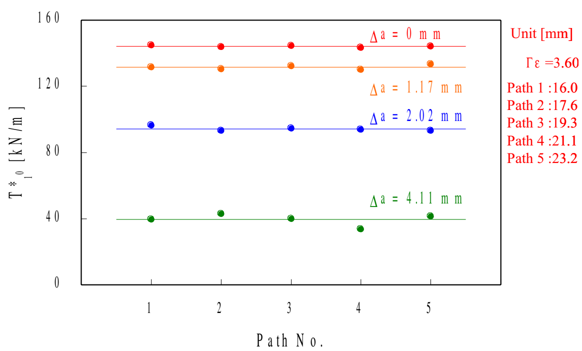


Figure 11: Independence of T^* integral

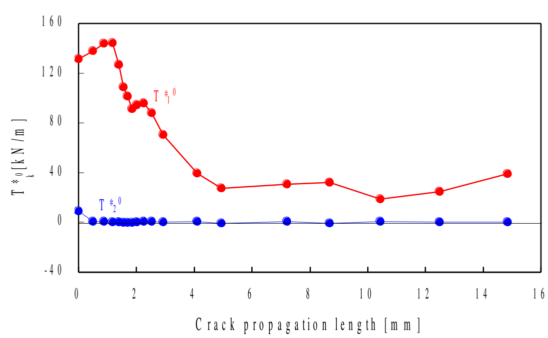


Figure 12: Histories of components of T^* integral

8 Conclusions

In this study a moving finite element method of elastic-plastic fracture was developed and numerical simulation for elastic-plastic fracture path prediction was achieved.

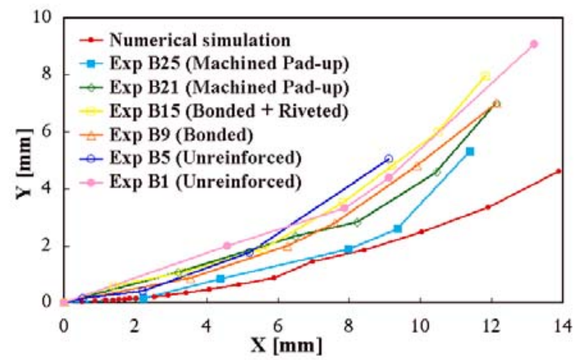


Figure 13: Predicted fracture path and experimental fracture paths

Predicted fracture path using $T^{*0}_{2} = 0$ criterion under mixed mode loading agrees with experimental fracture path.

Therefore, the moving finite element method is useful to estimate not only for elastic fracture problem but also for elastic-plastic fracture problem.

Acknowledgement: The authors acknowledge Prof. A. S. Kobayashi and his coworker, Dr. Lam for their variable advices and providing experimental data.

References

Atluri,S.N.; Nishioka,T.; Nakagaki,M.(1984): Incremental Path-Independent Integrals in Inelastic and Dynamic Fracture Mechanics. *Eng. Fract. Mech.*, **20**, 209-244

Brust, F. W.; Nishioka, T.; Atluri, S. N.; Nakagaki, M. (1985): Further Studies on Elastic-Plastic Stable Fracture Utilizing the T^*_ε Integral. Engineering Fracture Mechanics, Vol.22, No.6, pp.1079-1103.

Budiansky, B.; Rice, J. R. (1973): Conservation laws and energy release rate. J. Appl. Mech. 40 pp. 201-203.

Fujimoto, T.; Nishioka, T. (2006): Numerical Simulation of Dynamic Elasto Visco-plastic Fracture Using Moving Finite Element Method, *CMES: Computer Modeling in Engineering and Sciences*, Vol. 11, No. 2, pp. 91-102.

Kanninen, M. F. (1978): A critical appraisal of solution techniques in dynamic fracture mechanics, Numerical methods in fracture mechanics. (D.R.J.Owen and A.R. Luxmoore), *Pineridge Press* pp. 612-634.

Kobayashi, A. S.; Atluri, S. N. (1998): T* as a Static/Fatigue/Dynamic Crack Growth Parameter, Modelling and Simulation Based Engineering, Vol. II, eds. S.N. Atluri and P.E. O'Donoghue, *Tech Science Press*, 1305-1310.

Kobayashi, A. S.; Atluri, S. N. (2001): Experimental Validation of T* Integral, Proc. of ICF 10, eds: K. Ravi-Chandar, B.L. Karihaloo, T. Kishi, R.O. Richie, A.T. Yokobori and T. Yokobori, *Pergamon Press*..

Lam, P. W.; Kobayashi, A. S.; Atluri, S. N.; Tan, P. W. (1999): Further studies on T_{ε}^{*} integral for curved crack growth Mixed-mode crack behavior. *Proceedings of the Symposium, Atlanta, GA; UNITED STATES; 6-7 May 1998.* pp. 58-73.

Nishioka, T.; Atluri, S.N. (1983): Path-independent integrals, energy release rates, and general solutions of near-tip fields in mixed mode dynamic fracture mechanics. *Eng. Fract. Mech.*, 18, 1-22.

Nishioka, T. (1997): Computational Dynamic Fracture Mechanics. *International Journal of Fracture* 86 pp.127-159.

Nishioka, T.; Fujimoto, T. (2000): On the T* Integral and the Separated T* Integral. In: S.N. Atluri and F.W. Brust (ed) *Advances in Computational Engineering and Science* ,Volume II, pp.1866-1871.Tech Science Press.

Nishioka, T.; Tchouikov, S.; Fujimoto, T. (2001): Fracture Path Prediction Simulations of Dynamic Bifurcation Fracture Phenomena. In: S.N. Atluri, T. Nishioka and M. Kikuchi (ed) *Advances in Computational Engineering and Science*, Tech Science Press, chapter 7, paper 17, pp. 1-6.

Nishioka, T.; Tokudome, H.; Kinoshita, M. (2001): Dynamic Fracture-Path Prediction in Impact Fracture Phenomena Using Moving Finite Element Method Based on Delaunay Automatic Mesh Generation. *International Journal of Solids and Structure*, **38**, pp. 5273-5301.

Nishioka, T.; Furutsuka, J.; Tchouikov, S.; Fujimoto, T. (2002) Generation-Phase Simulation of Dynamic Crack Bifurcation Phenomenon Using Moving Finite Element Method based on Delaunay Automatic Triangulation. *CMES: Computer Modeling in Engineering and Sciences*, Vol.3, No.1, (2002), pp.129-145

Nishioka, T. (2005): Recent Advances in Numerical Simulation Technologies for Various Dynamic Fracture Phenomena. Workshop on Development and Advancement of Computational Mechanics, (In Celebrating Professor Satya N. Atluri's 60th Anniversary), (Invited Lecture) International Conference Center Kobe, April 22-23, (2005-4), *CMES: Computer Modeling in Engineering and Sciences*, Vol. 10, No.3, pp.209-215 (Special Issue)

Okada, H.; Atluri, S. N. (1999): Further studies on the characteristics of the T_{ε}^{*} integral: Plane stress stable crack propagation in ductile materials. *Computational Mechanics,*Vol.23, pp.339-352

Okada, H.; Atluri, S. N. (1997): A further study on the near tip integral parameter T_{ε}^{*} in stable crack propagation in thin ductile plate ; ASME Aerospace Division International Mechanical Engineering Congress and Exposition, Atlanta, GA; UNITED STATES; 17-22 Nov. 1996. pp. 251-260.

Sloan, S. W.; Houslby, G. T. (1984): An Implementation of Watson's Algorithm for Computing Two-dimensional Delaunay Triangulation. *Adv. in Eng. Soft.,* **6**, 192-197.

Taniguchi, T. (1992): Automatic Mesh Generation for FEM Use of Delaunay Triangulation. *Morikita Publishing*.