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Green Functions for a Continuously Non-homogeneous Saturated Media

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Abstract: An analytical solution is presented for the response of a non-homogeneous saturated poroelastic half-space under the action of a time-harmonic vertical point load on its surface. The shear modulus is assumed to increase continuously with depth and also the media is considered to obey Biot's poroelastic theory. The system of governing partial differential equations, based on the mentioned assumptions, is converted to ordinary differential equations' system by means of Hankel integral transforms. Then the system of equations is solved by use of generalized power series(Frobenius method) and the expressions for displacements in the interior of the media or in the other words, the Green functions for the media are derived by avoiding to introduction of any potential functions. Selected numerical results are presented to demonstrate the effect of depth non-homogeneity on dynamic response of the media.

keyword: Boundary element method, Green function, Depth non-homogeneity , Saturated media, Soil-structure interaction.

1 Introduction

Considering a constant depth profile for the shear modulus of soil in different soil-structure interaction problems is a rather poor approximation to the real sub-soil conditions since soil stiffness usually varies with depth in different layers of the soil.

In this paper, as shown in Fig. 1, an unbounded saturated media subjected to normal point load at the surface is considered. The mass density, porosity and permeability of the media are constant but the shear modulus varies solely with depth.

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The variation of shear modulus is described by an exponential function as follows [Selvadurai (1986)].

$$G(z) = G_\infty - (G_\infty - G_0)e^{-\alpha z} \tag{1}$$

Where G_0 and G_∞ are the shear modulus at the surface and infinite depth respectively and α is a constant with the dimension of inverse length, called coefficient of depth non-homogeneity or non-homogeneity parameter. By varying the parameters α, G_0 and G_∞, a wide range of real soil strata can be approximately described by Eq.1.

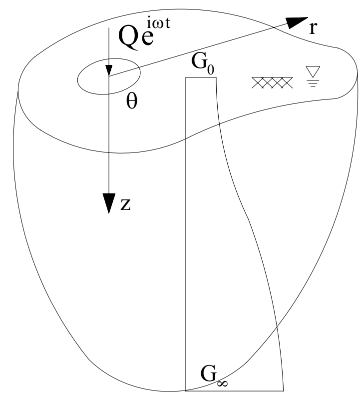

Figure 1 : Non-homogeneous saturated half space subjected to periodic normal point load at the surface

Firstly the system of governing differential equations, for the above media, obeying Biot's poro-elastic theory is derived. The system of equations, formed by four coupled partial differential equations, is converted to ordinary differential equations' system by means of Hankel integral transforms. Then the system of equations is solved by use of generalized power series (Frobenius method) and the expressions for displacements in the interior of the media or in the other words, the Green functions for the media are derived by avoiding to introduction of any potential functions. The results of the research can be approximately used to analyze the dynamic response of a multi-layer media and B.E.M. formulation for soil- structure interaction problems if the variation of shear modulus in different layers is estimated by an appropriate continuous function in the whole media.

2 Governing Differential Equations

Let *(r,θ,z)* be a cylindrical coordinate system, owing to the axisymmetric nature of the problem, the motions generated by the load configuration are independent of the angular coordinate θ and only displacements u and w in the $r-$ and $z-$directions, for the solid and fluid phases respectively, occur. So the equations of motion are:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=\rho\ddot{u}_r+\rho_f\ddot{w}_r \tag{2}$$

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\ddot{u}_z+\rho_f\ddot{w}_z \tag{3}$$

Where $\sigma_{ij}(i,j=r,\theta,z)$ are the components of the total stress-tensor, ρ denotes mass density of saturated media, ρ_f denotes mass density of the fluid phase and over dotes indicates derivatives respect to time. Also ρ, the mass density of the saturated media, is related to ρ_s, mass density of the solid phase and n, porosity, by the following formula:

$$\rho=(1-n)\rho_s+n\rho_f \tag{4}$$

Since the media obeys Biot's dynamic poroelastic theory, the following equations which are the mean of effective stress, dynamic equilibrium for the fluid phase (generalized Darcy law) and the mass conservation law respectively, could be written as[Biot (1956)].

$$\sigma'_{ij}=\sigma_{ij}+\delta_{ij}p \tag{5}$$

$$-\nabla p=\rho_f\ddot{u}+\frac{\alpha'\rho_f}{n}\ddot{w}+b\dot{w} \tag{6}$$

$$\dot{e}_s+\dot{e}_w+\frac{\dot{p}}{Q_f}=0 \tag{7}$$

where σ'_{ij} represents the components of effective stress tensor, P denotes the pore water pressure, δij is the Kroncker delta, α' denotes the additive mass coefficient, n denotes the porosity of saturated media, b denotes diffusive coefficient, Q_f denotes compressibility modulus of saturated media.

In the classic mechanics of porous media, the parameters α', b, and Q_f are defined as:

$$\alpha'=\frac{1}{2}(1+\frac{1}{n}) \tag{8}$$

$$b=\frac{g\rho_f n}{k'} \tag{9}$$

$$\frac{1}{Q_f}=\frac{n}{Q}+\frac{1-n}{k_s} \tag{10}$$

Where g is the acceleration of gravity, k' is the permeability of the media, Q is the compressibility modulus of the fluid phase and k_s is the bulk module of solid grains. Using the mean of effective stress, Eq.5, the equations of motion, Eq.2 and Eq.3 are described by the effective stress as follows:

$$\frac{\partial\sigma'_{rr}}{\partial r}+\frac{\partial\sigma'_{zr}}{\partial z}+\frac{\sigma'_{rr}-\sigma'_{\theta\theta}}{r}-\frac{\partial p}{\partial r}=\rho\ddot{u}_r+\rho_f\ddot{w}_r \tag{11}$$

$$\frac{\partial\sigma'_{rz}}{\partial r}+\frac{\partial\sigma'_{zz}}{\partial z}+\frac{\sigma'_{rz}}{r}-\frac{\partial p}{\partial z}=\rho\ddot{u}_z+\rho_f\ddot{w}_z \tag{12}$$

According to the linear elastic constitutive law, the stress-displacement relations can be written as:

$$\begin{aligned}
\sigma'_{rr}&=\lambda^*(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z})+2G^*\frac{\partial u_r}{\partial r}\\
\sigma'_{zz}&=\lambda^*(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z})+2G^*\frac{\partial u_z}{\partial z}\\
\sigma'_{\theta\theta}&=\lambda^*(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z})+2G^*\frac{u_r}{r}\\
\sigma'_{rz}&=G^*(\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial r})
\end{aligned} \tag{13}$$

where λ^* and G^* are complex Lame coefficients, for simplicity, the hysteretic type dissipation (frequency-independent) in the skeletal frame is assumed. It is further assumed that this dissipation is the same in bulk (volumetric) and shear straining. So λ^* and G^* are defined as:

$$\lambda^*=\lambda(1+2\delta i) \tag{14}$$

$$G^*=G(1+2\delta i)$$

Where δ is the hysteretic damping coefficient. The foregoing assumptions of course do not imply any restriction in the solution presented in this paper. In fact our solution is formulated in frequency-wave number domain thus any type of frequency dependent damping which may be different in bulk and shear straining is equally handled by the model.

Because the motion is time harmonic i.e.:

$$\begin{aligned} u_r(r,z,t) &= u_r(r,z)e^{i\omega t} \\ u_z(r,z,t) &= u_z(r,z)e^{i\omega t} \\ w_r(r,z,t) &= w_r(r,z)e^{i\omega t} \\ w_z(r,z,t) &= w_z(r,z)e^{i\omega t} \end{aligned} \tag{15}$$

The mass conservation law, Eq.7, using the definition of volumetric strain, can be written as:

$$p = -Q_f(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}) - Q_f(\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z}) \tag{16}$$

Using the above equation and the stress-displacement Eq.13, the equations of motions, Eq.11 and Eq.12, after some mathematical operation are converted to:

$$\begin{aligned} &(\overline{v}G^* + Q_f)(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{1}{r^2}u_r) + \rho\omega^2 u_r \\ &+ G^*\frac{\partial^2 u_r}{\partial z^2} + \frac{\partial G^*}{\partial z}\frac{\partial u_r}{\partial z} + (\frac{G^*}{1-2v} + Q_f)\frac{\partial^2 u_z}{\partial r\partial z} \\ &+ \frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial r} + Q_f(\frac{\partial^2 w_r}{\partial r^2} + \frac{1}{r}\frac{\partial w_r}{\partial r} - \frac{1}{r^2}w_r \\ &+ \frac{\partial^2 w_z}{\partial r\partial z}) + \rho_f\omega^2 w_r = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} &(\frac{G^*}{1-2v} + Q_f)(\frac{\partial^2 u_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z}) \\ &+ \frac{2v}{1-2v}\frac{\partial G^*}{\partial z}(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}) + (\overline{v}G^* + Q_f)\frac{\partial^2 u_z}{\partial z^2} \\ &+ G^*(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}) + \overline{v}\frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial z} + \rho\omega^2 u_z \\ &+ Q_f(\frac{\partial^2 w_r}{\partial z\partial r} + \frac{1}{r}\frac{\partial w_r}{\partial z} + \frac{\partial^2 w_z}{\partial z^2}) + \rho_f\omega^2 w_z = 0 \end{aligned} \tag{18}$$

where

$$\overline{\nu} = \frac{2(1-v)}{1-2v} = \frac{\lambda^* + 2G^*}{G^*} \tag{19}$$

on the other hand, using Eq.16, the dynamic equilibrium for the fluid phase (generalized Darcy law), Eq.6, is converted to the following equations after some mathematical operations:

$$\begin{aligned} &Q_f(\frac{\partial^2 u_r}{\partial r^2} - \frac{u_r}{r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_z}{\partial r\partial z}) + Q_f(\frac{\partial^2 w_r}{\partial r^2} - \frac{w_r}{r^2} \\ &+ \frac{1}{r}\frac{\partial w_r}{\partial r} + \frac{\partial^2 w_z}{\partial r\partial z}) + \rho_f\omega^2 u_r + \frac{\alpha\rho_f}{n}\omega^2 w_r - ib\omega w_r = 0 \end{aligned} \tag{20}$$

$$\begin{aligned} &Q_f(\frac{\partial^2 u_r}{\partial r^2} + \frac{\partial u_r}{r\partial z} + \frac{\partial^2 u_z}{\partial z^2}) + Q_f(\frac{\partial^2 w_r}{\partial r^2} + \frac{\partial w_r}{r\partial z} + \frac{\partial^2 w_z}{\partial z^2}) \\ &+ \rho_f\omega^2 u_z + \frac{\alpha'\rho_f}{n}\omega^2 w_r - ib\omega w_z = 0 \end{aligned} \tag{21}$$

The boundary conditions of problem are:

$$\begin{aligned} &@z=0 \quad \sigma'_{zz}(r,z) = \lambda^*(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}) \\ &+2G^*\frac{\partial u_z}{\partial z} = Q\delta(r) \end{aligned} \tag{22}$$

$$@z=0 \quad \sigma'_{rz}(r,z) = G^*(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}) = 0 \tag{23}$$

$$\begin{aligned} &@z=0 \quad p = -Q_f((\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}) \\ &+(\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z})) = 0 \end{aligned} \tag{24}$$

Where δ is the Dirac delta function.

In addition, the solution must be such that the stresses and displacements are bounded at a remote distance and only outward waves propagating from the source appear (radiation condition).

The four coupled second-order partial differential equations i.e. Eqs. (17),(18),(20) and (21) subjected to above boundary conditions defines the boundary value problem for the response of a saturated non-homogeneous half-space to the vertical surface load.

As it is mentioned before, the derive of above equations, is based on axisymmetric nature of the problem, which is due to the action of normal point load at the surface. So the model's implementation is limited to vertical point load, not for the other loading systems such as horizontal loading.

3 General Solution

The general solution of the system of governing differential equations can be obtained by employing a Hankel transforms for the radial coordinate r. So we have:

$$\begin{aligned} H_1(u_r(r,z)) &= \overline{u}_r(k,z) = \int_{\circ}^{+\infty} r u_r(r,z) J_1(kr)dr \\ H_{\circ}(u_z(r,z)) &= \overline{u}_z(k,z) = \int_{\circ}^{+\infty} r u_z(r,z) J_{\circ}(kr)dr \\ H_1(w_r(r,z)) &= \overline{w}_r(k,z) = \int_{\circ}^{+\infty} r w_r(r,z) J_1(kr)dr \\ H_{\circ}(w_z(r,z)) &= \overline{w}_z(k,z) = \int_{\circ}^{+\infty} r w_z(r,z) J_{\circ}(kr)dr \end{aligned} \tag{25}$$

Where k is the Hankel transforms parameter and J_n is the Bessel function of the first kind of order n. Substituting of above equations in Eqs. (17),(18),(20) and (21) and making use of the expressions for Hankel transform of the derivatives of a function yields:

$$\begin{aligned} &G^*\frac{\partial^2\overline{u}_r}{\partial z^2} + \frac{\partial G^*}{\partial z}\frac{\partial \overline{u}_r}{\partial z} + (\rho\omega^2 - k^2(\overline{v}G^* + Q_f))\overline{u}_r \\ &\quad - k(G^*(\overline{v}-1) + Q_f)\frac{\partial \overline{u}_z}{\partial z} - k\frac{\partial G^*}{\partial z}u_z \\ &\quad + (\rho_f\omega^2 - k^2 Q_f)\overline{w}_r - Q_f k\frac{\partial \overline{w}_z}{\partial z} = 0 \end{aligned} \tag{26}$$

$$\begin{aligned} &((\overline{v}-1)G^* + Q_f)k\frac{\partial \overline{u}_r}{\partial z} + (\overline{v}-2)\frac{\partial G^*}{\partial z}k\overline{u}_r + (\overline{v}G^* \\ &\quad + Q_f)\frac{\partial^2\overline{u}_z}{\partial z^2} + \overline{v}\frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial z} + (\rho\omega^2 - k^2 G^*)\overline{u}_z \\ &\quad + kQ_f\frac{\partial \overline{w}_r}{\partial z} + Q_f\frac{\partial^2 w_z}{\partial z^2} + \rho_f\omega^2\overline{w}_z = 0 \end{aligned} \tag{27}$$

$$\begin{aligned} &(\rho_f\omega^2 - k^2 Q_f)\overline{u}_r - kQ_f\frac{\partial \overline{u}_z}{\partial z} + (\frac{\alpha\rho_f}{n}\omega^2 - k^2 Q_f - ib\omega)\overline{w}_r \\ &\quad - kQ_f\frac{\partial \overline{w}_z}{\partial z} = 0 \end{aligned} \tag{28}$$

$$\begin{aligned} &kQ_f\frac{\partial \overline{u}_r}{\partial z} + Q_f\frac{\partial^2\overline{u}_z}{\partial z^2} + \rho_f\omega^2\overline{u}_z + kQ_f\frac{\partial \overline{w}_r}{\partial z} + \\ &\quad Q_f\frac{\partial^2\overline{w}_z}{\partial z^2} + (\frac{\alpha\rho_f}{n}\omega^2 - ib\omega)\overline{w}_z = 0 \end{aligned} \tag{29}$$

if a subsidiary depth variable is introduced as follows:

$$\xi = E_0 e^{-\alpha z} \tag{30}$$

Where

$$E_0 = 1 - \frac{G_0^*}{G_\infty^*} \tag{31}$$

Which transforms the interval $0 \le z \le H$ onto $E \ge \xi \ge 0$, then shear modulus variation, Eq.1, reduces to:

$$G^* = G_\infty^*(1-\xi) \tag{32}$$

E_0 can be regarded as a measure of the non-homogeneity of the half space medium. $E_0 \rightarrow 0$ corresponds to the homogeneous half space ($G_0 \rightarrow G_\infty$).

Inserting the above transformations into the differential equations i.e. Eqs. (26)-(29) results in:

$$\begin{aligned} &\alpha^2\xi^3\overline{u}_r'' - \alpha^2\xi^2\overline{u}_r'' + 2\alpha^2\xi^2\overline{u}_r' - \alpha^2\xi\overline{u}_r' - k^2\overline{v}\xi\overline{u}_r \\ &\quad - (\frac{\rho\omega^2}{G_\infty^*} - k^2\overline{v} - \frac{k^2 Q_f}{G_\infty^*})\overline{u}_r + k\alpha(\overline{v}-1)\xi^2\overline{u'}_z \\ &\quad - (k\alpha(\overline{v}-1) + \frac{k\alpha Q_f}{G_\infty^*})\xi\overline{u'}_z + k\alpha\xi\overline{u}_z \\ &\quad - (\frac{\rho_f\omega^2}{G_\infty^*} - \frac{k^2 Q_f}{G_\infty^*})\overline{w}_r - \frac{\alpha k Q_f}{G_\infty^*}\xi\overline{w'}_z = 0 \end{aligned} \tag{33}$$

$$\begin{aligned} &-\alpha k(\overline{v}-1)\xi^2\overline{u'}_r + (k\alpha(\overline{v}-1) + \frac{k\alpha Q_f}{G_\infty^*})\xi\overline{u'}_r - \alpha k(\overline{v}-2)\xi\overline{u}_r \\ &\quad + \overline{v}\alpha^2\xi^3\overline{u''}_z + (\alpha^2\overline{v} + \frac{\alpha^2 Q_f}{G_\infty^*})\xi^2\overline{u''}_z + 2\overline{v}(\alpha^2\xi^2)\overline{u'}_z \\ &\quad - (\alpha^2\overline{v} + \frac{\alpha^2 Q_f}{G_\infty^*})\xi\overline{u'}_z - k^2\xi u_z - (\frac{\rho\omega^2}{G_\infty^*} - k^2)\overline{u}_z + \frac{kQ_f\alpha}{G_\infty^*}\xi\overline{w'}_r \\ &\quad - \frac{Q_f\alpha^2}{G_\infty^*}\xi^2\overline{w''}_z - \frac{Q_f\alpha^2}{G_\infty^*}\xi\overline{w'}_z - \frac{\rho_f\omega^2}{G_\infty^*}\overline{w}_z = 0 \end{aligned} \tag{34}$$

$$\begin{aligned} &(\rho_f\omega^2 - k^2 Q_f)\overline{u}_r + \alpha k Q_f\xi\overline{u}_z' + \alpha k Q_f\xi\overline{w}_z' \\ &\quad + (\frac{\alpha'\rho_f}{n}\omega^2 - k^2 Q_f - ib\omega)\overline{w}_r' = 0 \end{aligned} \tag{35}$$

$$\begin{aligned} &-\alpha k Q_f\xi\overline{u}_r' + Q_f\alpha^2\xi^2\overline{u}_z'' + \alpha^2 Q_f\xi\overline{u}_z' + \rho_f\omega^2\overline{u}_z \\ &\quad - \alpha k Q_f\xi\overline{w'}_r + Q_f\alpha^2\xi^2\overline{w}_z'' + Q_f\alpha^2\xi\overline{w}_z' \\ &\quad + (\frac{\alpha'\rho_f}{n}\omega^2 - ib\omega)\overline{w}_z = 0 \end{aligned} \tag{36}$$

It is necessary to rewrite the boundary conditions by using subsidiary depth variable and Hankel Integral variables. So the boundary conditions, Eqs.(22)-(24) are converted to:

$$@\xi = E_{\circ} \quad (\overline{v}-2)k\overline{u}_r + \overline{v}(-\alpha\xi)\frac{d\overline{u}_z}{d\xi} = \frac{Q}{2\pi G_0^*} \tag{37}$$

$$@\xi = E_\circ \quad (-\alpha\xi)\frac{d\overline{u}_r}{d\xi} - k\overline{u}_z = 0 \tag{38}$$

$$@\xi = E_\circ \quad k\overline{u}_r + (-\alpha\xi)\frac{d\overline{u}_z}{d\xi} + k\overline{w}_r + (-\alpha\xi)\frac{d\overline{w}_z}{d\xi} = 0 \tag{39}$$

Analytical solutions for the system of differential equations (33)-(36) can be found by using the Frobenius method (extended power series method). According to the method, the general solution are given by a linear combination of power series as follows[Boyce and Diprima (1992)].

$$\overline{u}_r = \sum_{i=1}^{6} A_i(k) \sum_{n=0}^{\infty} a_n \xi^{n+m_i} \tag{40}$$

$$\overline{u}_z = \sum_{i=1}^{6} A_i(k) \sum_{n=0}^{\infty} b_n \xi^{n+m_i} \tag{41}$$

$$\overline{w}_r = \sum_{i=1}^{6} A_i(k) \sum_{n=0}^{\infty} c_n \xi^{n+m_i} \tag{42}$$

$$\overline{w}_z = \sum_{i=1}^{6} A_i(k) \sum_{n=0}^{\infty} d_n \xi^{n+m_i} \tag{43}$$

where m_i (i=1 to 6) are complex roots of the following equations:

$$Det\begin{bmatrix} -(\alpha^2 m^2 + (\frac{\rho\omega^2}{G^*_\infty} - k^2\overline{\nu} - \frac{k^2 Q_f}{G^*_\infty})) & -(k\alpha(\overline{\nu}-1) + \frac{kQ_f\alpha}{G^*_\infty})m & -(\frac{\rho_f\omega^2}{G^*_\infty} - \frac{k^2 Q_f}{G^*_\infty}) & -\frac{k\alpha Q_f}{G^*_\infty}m \\ (k\alpha(\overline{\nu}-1) + \frac{k\alpha Q_f}{G^*_\infty})m & (-(\overline{\nu}\alpha^2 + \frac{\alpha Q_f}{G^*_\infty})m^2 + (\frac{\rho_f\omega^2}{G^*_\infty} - k^2)) & \frac{kQ_f\alpha}{G^*_\infty}m & -(\frac{Q_f\alpha}{G^*_\infty}m^2 + \frac{\rho_f\omega^2}{G^*_\infty}) \\ \rho_f\omega^2 - k^2 Q_f & k\alpha Q_f m & \frac{\alpha\rho_f}{n}\omega^2 - k^2 Q_f - ib\omega & \alpha k Q_f m \\ -\alpha k Q_f m & Q_f\alpha^2 m^2 + \rho_f\omega^2 & -\alpha k Q_f m & Q_f\alpha^2 m^2 + (\frac{\alpha\rho_f}{n} - ib\omega) \end{bmatrix} = 0 \tag{44}$$

and can be described as:

$$\begin{aligned} m_1 &= +R + Ii \\ m_4 &= -R - Ii \\ m_2 &= R' + I'i \\ m_5 &= -R' - I'i \\ m_3 &= R'' + I''i \\ m_6 &= -R'' - I''i \\ I, I', I'' &> 0 \end{aligned} \tag{45}$$

And $A_i(k)$ are arbitrary functions to be determined from appropriate boundary conditions. The coefficients of power series are determined by considering:

$$a_0^i = 1 \quad (i = 1 to 6) \tag{46}$$

and solving simultaneously the following system of equations for b_0, c_0 and d_0:

$$\begin{aligned} &-(\alpha^2 m^2 + (\frac{\rho\omega^2}{G^*_\infty} - k^2\overline{\nu} - \frac{k^2 Q_f}{G^*_\infty}))a_\circ - (k\alpha(\overline{\nu}-1) \\ &\quad + \frac{k\alpha Q_f}{G^*_\infty})mb_\circ - (\frac{\rho_f\omega^2}{G^*_\infty} - \frac{k^2 Q_f}{G^*_\infty})c_\circ - (\frac{k\alpha Q_f}{G^*_\infty}m)d_\circ = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} &(k\alpha(\overline{\nu}-1) + \frac{k\alpha Q_f}{G^*_\infty})ma_\circ \\ &\quad - ((\overline{\nu}\alpha^2 + \frac{\alpha^2 Q_f}{G^*_\infty})m^2 + (\frac{\rho_f\omega^2}{G^*_\infty} - k^2))b_\circ \\ &\quad + \frac{kQ_f\alpha}{G^*_\infty}c_\circ m - (\frac{Q_f\alpha^2}{G^*_\infty}m^2 + \frac{\rho_f\omega^2}{G^*_\infty})d_\circ = 0 \end{aligned} \tag{48}$$

$$\begin{aligned} &(\rho_f\omega^2 - k^2 Q_f)a_\circ + k\alpha Q_f b_\circ m + (\frac{\alpha'\rho_f}{n}\omega^2 \\ &\quad - k^2 Q_f - ib\omega)c_\circ + \alpha k Q_f m d_\circ = 0 \end{aligned} \tag{49}$$

4 Solution of Boundary Value Problem

Attention to definition of subsidiary depth variable, Eq. 30, it could be seen that the first three terms of equations (40)-(43) in conjunction with time behavior, *exp($i\omega t$)* gives outward–propagating waves. Consequently, these terms should be retained and the other terms should be omitted to satisfy the radiation condition. So their arbitrary coefficients set equal to zero:

$$A_4 = A_5 = A_6 = 0 \tag{50}$$

The remaining three arbitrary functions $A_1(k)$,*$A_2(k)$ and* $A_3(k)$ are determined by three boundary conditions at the surface, equations (37),(38) and (39). Substituting the available general solution, Eqs. (40) to (43) into mentioned boundary conditions, we have:

$$\begin{aligned} &(\overline{\nu}-2)k\sum_{i=1}^{3} A_i \sum_{n=0}^{\infty} a_n^i E_\circ^{n+m_i} + \overline{\nu}(-\alpha) \\ &\quad \sum_{i=1}^{3} A_i \sum_{n=0}^{\infty} (n+m_i) b_n^i E_\circ^{n+m_i} = \frac{Q}{2\pi G^*_0} \end{aligned} \tag{51}$$

Table 1 : Selected models' properties

No.	Models' Parameter	Value	Unit
1	Shear modulus at surface (G_0)	54	MPa
2	Compressibility modulus of fluid (Q)	2068	MPa
3	Bulk modulus of fluid (k_s)	35000	Mpa
4	Mass density of grains (ρ_s)	2650	Kg/m^3
5	Mass density of fluid (ρ_f)	1000	Kg/m^3
6	Permeability(k')	3*10^{-7}	m/s
7	Porosity(n)	0.3	---
8	Poisson's ratio(v)	0.3	---
9	Hysteretic damping(δ)	0.05	---

$$(-\alpha)\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}(n+m_i)a_n^iE_\circ^{n+m_i}-k\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}b_n^iE_\circ^{n+m_i}=0 \tag{52}$$

$$k\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}a_n^iE_\circ^{n+m_i}+(-\alpha)\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}(n+m_i)b_n^iE_\circ^{n+m_i}$$
$$+k\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}c_n^iE_\circ^{n+m_i}$$
$$+(-\alpha)\sum_{i=1}^{3}A_i\sum_{n=0}^{\infty}(n+m_i)d_n^iE_\circ^{n+m_i}=0 \tag{53}$$

Solving the above system of simultaneous equations, leads to determination of $A_1(k), A_2(k)$ *and* $A_3(k)$ and using them into Eqs.(40) to (43) results explicit solution for displacement function at any point within the media in the $\omega-k$ domain.

5 Results

The presented analytical solution in the previous sections has been applied to investigate the depth non-homogeneity effect on dynamic response of the media and Green functions utilizing the dimensionless variables as follows:

$$\overline{Re}(u_z)=Re(u_z).k \tag{54}$$

$$\overline{Re}(u_r)=Re(u_r).k \tag{55}$$

$$\overline{\varpi}={}^{c}/_{V_s} \tag{56}$$

$$\overline{\alpha}=\alpha/k \tag{57}$$

Where $\overline{\varpi}$ is dimensionless frequency, $\overline{\alpha}$ is dimensionales non-homogeneity parameter $\overline{Re}(u_z)$ and $\overline{Re}(u_r)$ are dimensionless real parts of vertical and radial surface displacements respectively. V_s and c are defined as follows:

$$V_s=\sqrt{\frac{G_0}{\rho}} \tag{58}$$

$$c=\frac{\omega}{k} \tag{59}$$

In order to study the effect of depth non-homogeneity on the response of the media, the variation of dimensionless real parts of vertical and radial displacements versus dimensionless frequency are illustrated in figures 2 and 3 for different values of G_0/G_∞ ratio and different values of depth non-homogeneity parameter in each figure. The model's properties are given in Tab.1.

Also in Fig.4 and Fig.5, Three dimensional diagrams of Real part of vertical displacement are drawn to show clearly how the displacement field is dependent to non-homogeneity parameters i.e. $\frac{G_0}{G_\infty}$ and α.

To validate the formula derived above, the presented solution is computed for the surface displacement of the half space when $G_0/G_\infty \rightarrow 1$ and then compared with the solution which is available in the literature for the homogeneous poroelastic state. The comparison is shown in Fig. 6. The agreement of the results is found to be excellent [Philippacopoulos (1988)]. It is necessary to note

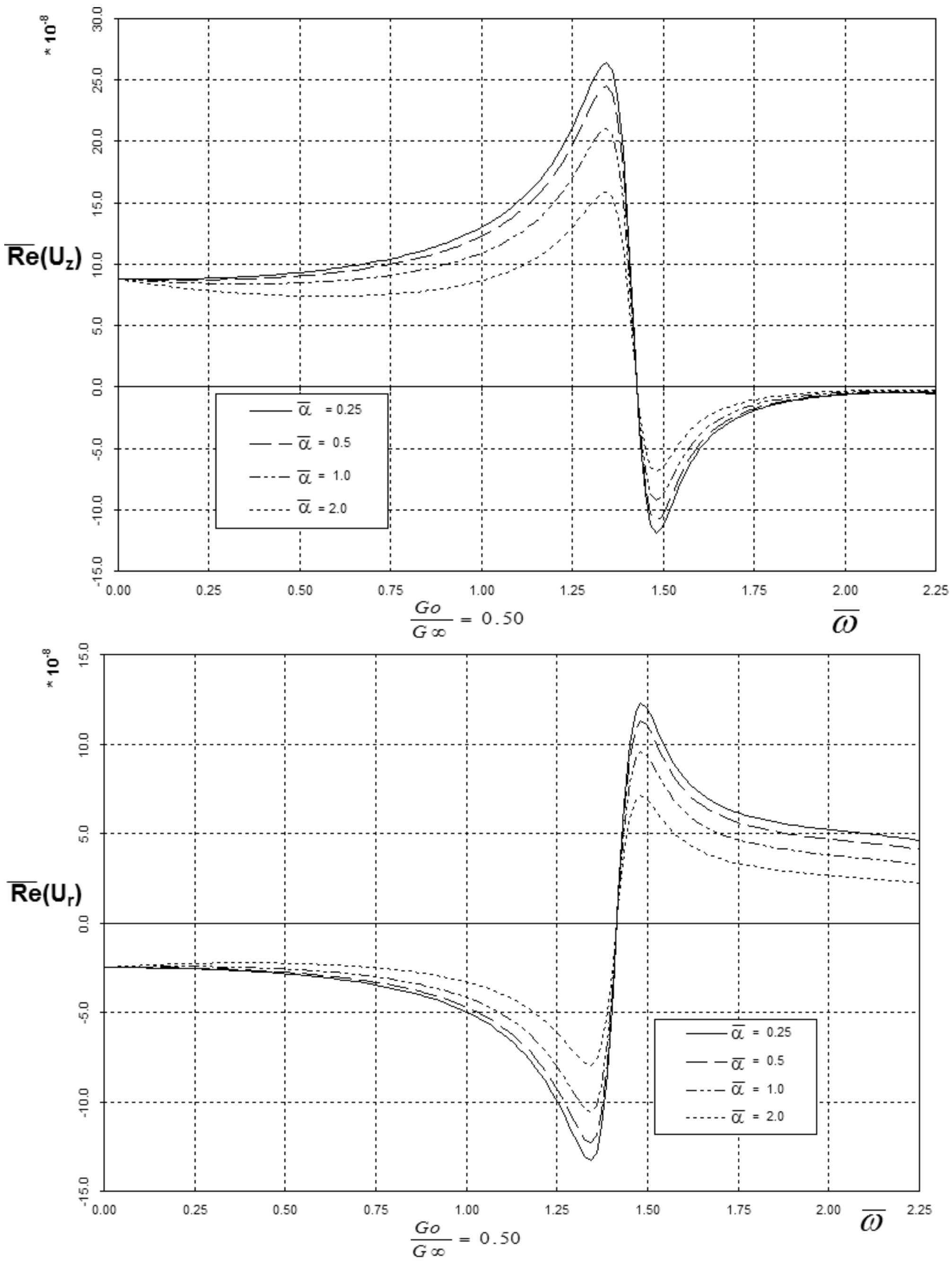

Figure 2 : Variation of vertical and radial displacements versus frequency for different depth non-homogeneity parameters ($\frac{G_0}{G_\infty} = 0.5$)

that when $G_0/G_\infty \rightarrow 1$, the non-homogeneity parameter, α has no longer effect.

6 Conclusion

The solution of the response of a non-homogeneous saturated elastic half space media to a periodic vertical surface point load has been presented. By avoiding the introduction of potential and choosing exponential func-

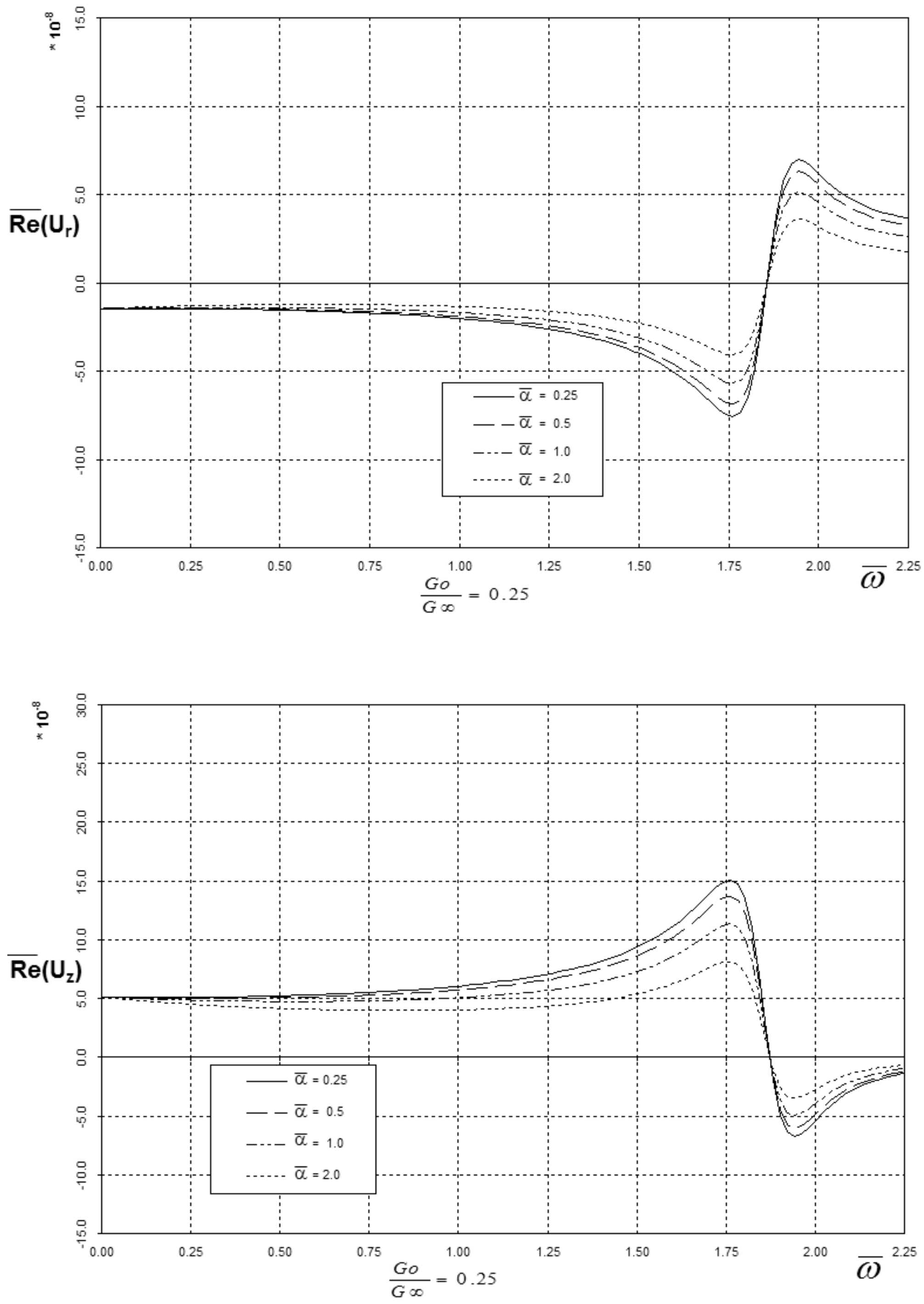

Figure 3 : Variation of vertical and radial displacements versus Frequency for diffrend depth non-homogeneity parameters ($\frac{G_0}{G_\infty} = 0.25$)

tion for the shear modulus depth–variation, the boundary value problem can be solved by use of Hankel integral transform and applying extended power series method. Selected numerical results including the variation of surface displacements versus frequency for different values of depth non-homogeneity parameter show the dynamic

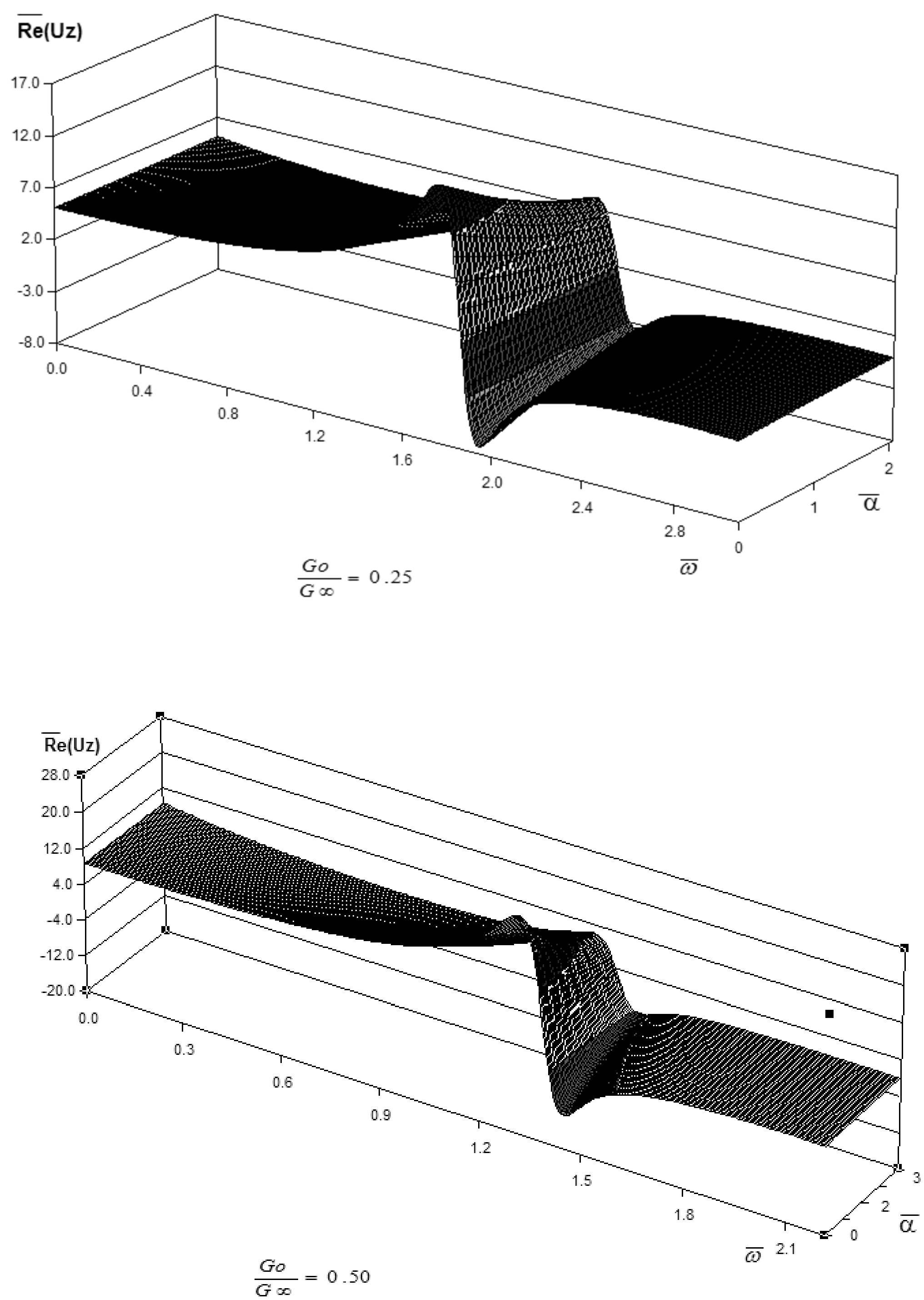

Figure 4 : Variation of dimensionless real part of vertical displacement versus frequency and non-homogeneity parameter ($\frac{G_0}{G_\infty} = 0.25\&0.5$)

response of the media and Green functions are strongly dependant to shear modulus distribution and depth non-homogeneity.

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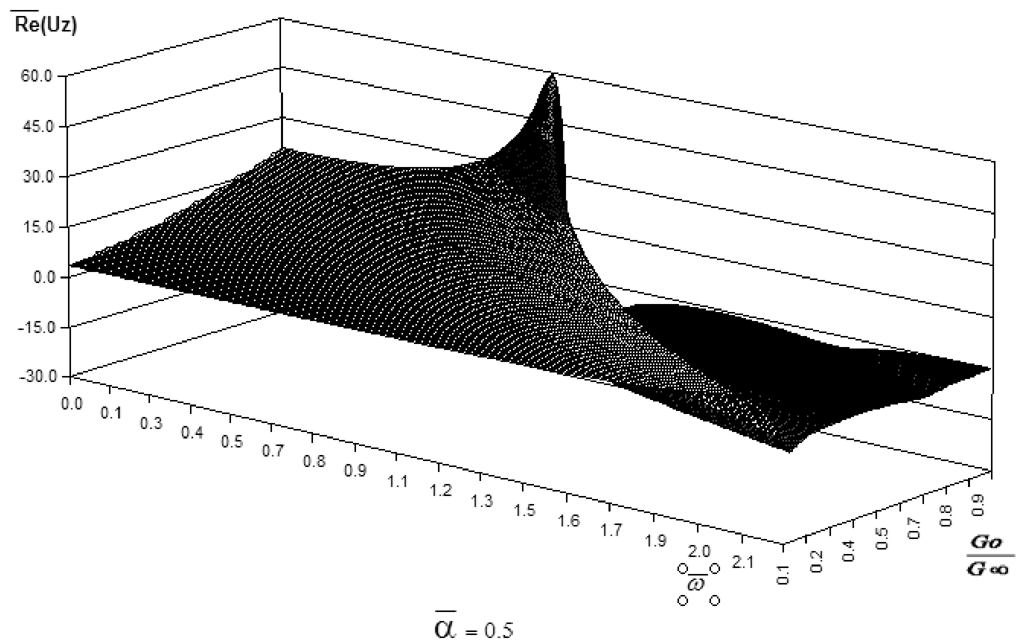

Figure 5 : Variation of dimensionless real part of vertical displacement versus frequency and shear modulus ratio $(\alpha = 0.5)$

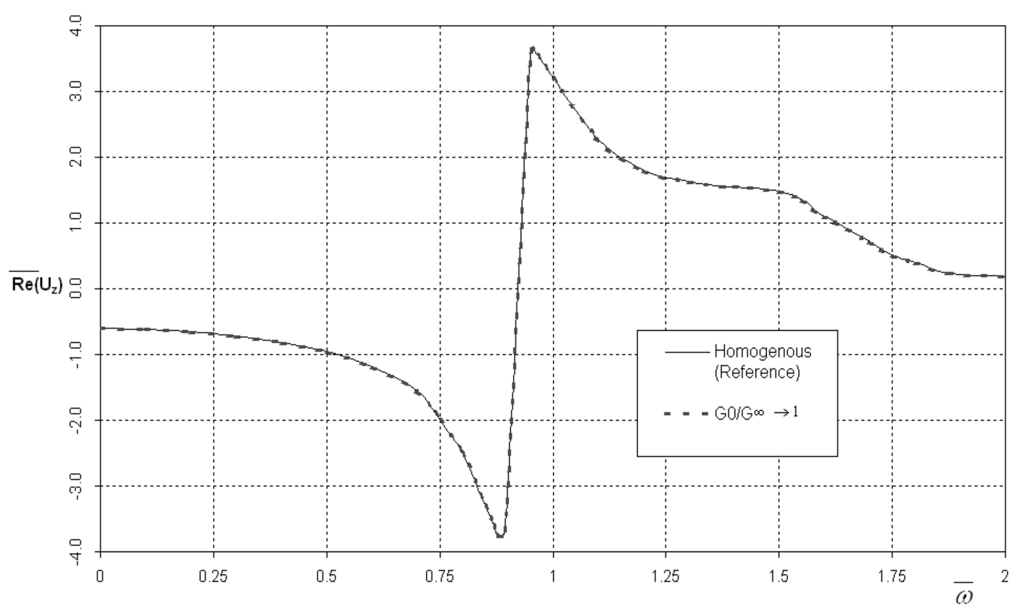

Figure 6 : Comparison of presented numerical solution when $G_0/G_\infty \to 1$ and the available solution for the homogeneous state

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