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An Efficient Time-Domain BEM/FEM Coupling for Acoustic-Elastodynamic Interaction Problems

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Abstract: A coupling procedure is described to perform time-domain numerical analyses of dynamic fluid-structure interaction. The fluid sub-domains, where acoustic waves propagate, are modeled by the Boundary Element Method (BEM), which is quite suitable to deal with linear homogeneous unbounded domain problems. The Finite Element Method (FEM), on the other hand, models the structure sub-domains, adopting a time marching scheme based on implicit Green's functions. The BEM/FEM coupling algorithm here developed is very efficient, eliminating the drawbacks of standard and iterative coupling procedures. Stability and accuracy features are improved by the adoption of different time steps in each sub-domain of the coupled analysis.

keyword: BEM/FEM coupling; fluid-structure interaction; acoustics; elastodynamics; Green's functions; adjustable time steps.

1 Introduction

Accurate modelling of infinite domain problems with the Finite Element Method (FEM) – and other domain discretization methods, e.g., Finite Differences, Finite Volumes etc. – has been the subject of intense investigation since the method started being employed to solve practical engineering (and other fields) problems. Infinite elements and other similar schemes have been successfully employed in some time-independent problems; however, in time-dependent problems the success is conditioned to the correct choice of one or more calibration parameters. The very first researches, that employed the FEM to model either harmonic or arbitrarily time-dependent wave propagation problems, were faced with artificial boundary reflections, which can invalidate the numerical response [Bettess and Bettess (1991), Givoli (1991)].

Methods based on plane wave approximations (PWA) were among the first appearing in the literature to simulate the effect of the infinite media [Mindlin and Bleich (1952), DiMaggio, Sandler and Rubin (1981), Hamdan and Dowling (1995)]. PWA is a very simple and elegant approach, however, it only carries away part of the energy supposed to be dissipated through the infinite medium (the knowledge of the direction of the wave hitting the transmitting boundary may improve the PWA accuracy). Although the coupling of PWA with the virtual mass approximation (VMA) method [Geers (1969)], as proposed by Ranlet et al. (1977) improved the performance of PWA, it is presently accepted that methods based on plane wave approximations (and other simplified approaches) can work well only for the far field. Other semi-analytical approaches, which work quite well for some particular cases (e.g., coupling of structures with stratified fluid or elastic infinite media), have been employed to simulate energy dissipation by infinite media [Wolf (1985), Kausel (1994)].

Many different approaches have been developed after the PWA models, some of them have been reported as good "transmitting boundaries" ("silent boundaries" is also a common designation adopted); the Boundary Element Method (BEM) is one of the most successful. BEM is commonly used in both time-domain and frequency-domain algorithms. As examples of BEM/FEM coupling in frequency domain the reader is referred to Jeans and Mathews (1990), Everstine and Henderson (1990), Tanaka et al. (1998), Tadeu and Godinho (2003) etc..

The first algorithms, which employed BEM/FEM coupling, followed a standard coupling approach (SCA) where BEM and FEM equations are coupled into a unique system of algebraic equations through direct application of equilibrium and compatibility conditions to the interface between FEM and BEM domains. The first applications of SCA in time-domain were con-

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cerned with soil-structure interactions [Karabalis and Beskos (1985), Spyrakos and Beskos (1986), Estorff and Prabucki (1990)]. Soon after these first works, Estorff and Antes (1991) presented the extension of the formulation presented by Estorff and Prabucki (1990) to consider fluid-structure interaction and, recently, non-linear modelling has been considered within the FEM sub-domain [Czygan and Estorff (2002)]. As it has been reported [Yu, Lie and Fan (2002), Czygan (2002)], SCA leads to several problems with respect to accuracy, efficiency and stability. Some authors have improved the stability and accuracy features of the time-domain BEM approach, proposed by Mansur (1983) [see also Mansur and Brebbia (1982a), Mansur and Brebbia (1982b)]: improved shape functions [Frangi (2000)], alternative time marching processes [Yu, Mansur, Carrer and Lei (1998)], variational approaches [Belytschko and Lu (1994)] etc. have been developed. In fact, Yu et al. (2001) used the linear θ method and the SCA in time-domain fluid-structure and structure-structure interaction problems and reported great stability improvement. Stability improvements in SCA have also been reported by Belytschko and Lu (1994) and can be expected if the technique worked out by Frangi (2000) is employed. In fact, several techniques [Yan, Cui and Hung (2005), Callsen, Estorff and Zaleski (2004), Qian, Han, Ufimtsev and Atluri (2004), Qian, Han and Atluri (2004)] may be adopted in order to improve specific points on the formulation.

Iterative coupling approaches [Elleithy, Al-Gahtani and El-Gebeily (2001)] (ICA) allow BEM and FEM domains to be treated separately, leading to smaller and better conditioned systems of equations to be solved and, in the case of time-domain analyses, one may employ different time steps for each sub-domain. Consideration of different time steps for BEM and FEM improves substantially stability and accuracy. Additional improvements can be obtained if the very same procedures already developed for time-domain FEM, BEM and/or BEM/FEM SCA are employed to the FEM/BEM ICA. In previous publications [Soares Jr and Estorff (2004), Soares Jr, Estorff and Mansur (2004), Soares Jr, Estorff and Mansur], the authors highlighted the improvement of the stability and accuracy of the ICA, when compared to the SCA, due to the possibility of considering different time-steps for each sub-domain. Computational efficiency improvement has also been reported: (a) few iterations are required for the ICA convergence and they can be carried out together with non-linear analysis; (b) different solvers, suitable for each sub-domain can be employed; (c) the system of equations is smaller than that obtained by SCA algorithms; (d) BEM equations do not need to be dealt with in each step of the non-linear process; etc.

The present paper presents an even more efficient BEM/FEM coupling approach for time-domain fluid-structure interaction analyses. A time marching algorithm based on FEM implicit Green's functions [Soares Jr and Mansur (2005)] is employed to model the solid sub-domain (structure or continuous). In this way, two major advantages are achieved: (a) FEM and BEM sub-domains are solved separately; (b) coupling iterations, as required by the ICA, become unnecessary. Fluid sub-domains are here modelled by the standard time-domain BEM formulation [Mansur (1983),Dominguez (1993)].

The present text is organized as follows: firstly the adopted BE and FE formulations are shown and briefly discussed (section 2 and 3). In the sequence, the proposed BEM/FEM coupling procedure is presented (section 4), followed by some numerical examples (section 5). At the end of the paper, conclusions and general remarks are presented, highlighting the potentialities of the new procedure.

2 Modelling the BEM sub-domain

The governing equation for a homogeneous acoustic fluid can be written as

$$\nabla^2 p(X,t) - \frac{1}{c^2}\ddot{p}(X,t) = -\widehat{s}(X,t) \tag{1}$$

where ∇^2 is the Laplacian operator; p is the transient fluid pressure; $\ddot{p}$ is the second partial time derivative of p; $\widehat{s}$ is the source density and c is the sound speed ($c=(K/\rho)^{1/2}$, where K and ρ are the fluid compression modulus and density, respectively). The integral equation which solves equation (1), can be written as

$$\begin{aligned} 4\pi c(\xi)\,p(\xi,t) &= \rho \\ &-\Big(\int_\Gamma\int_0^{t+} p^*(X,t;\xi,\tau)\,\ddot{u}_N(X,\tau)\,d\tau\, d\Gamma(X) \\ &+\int_\Gamma\int_0^{t+} \ddot{u}_N^*(X,t;\xi,\tau)\,p(X,\tau)\,d\tau\, d\Gamma(X)\Big) \\ &+\int_\Omega\int_0^{t+} p^*(x,t;\xi,\tau)\,\widehat{s}(x,\tau)\,d\tau\, d\Omega(x) \end{aligned} \tag{2}$$

In the above equation, the following fundamental solutions are employed

$$p^*(X,t;\xi,\tau) = \frac{2c\,H\,[\,c\,(t-\tau)-r\,]}{[\,c^2(t-\tau)^2-r^2\,]^{1/2}} \tag{3}$$

$$\ddot{u}^*_N(X,t;\xi,\tau) = -\frac{1}{\rho}\left(\frac{\partial p^*}{\partial N}\right) \tag{4}$$

where$H\,[c\,(t-\tau)-r]$ stands for the Heaviside function; $r = r(X;\xi)$ is the distance between the field point (X) and the source point (ξ); and N stands for the co-ordinate in the direction of outward vector, normal to Γ at X. $\ddot{u}_N = -(1/\rho)(\partial p/\partial N)$ is the normal acceleration on the boundary.

In order to implement a numerical scheme for the two-dimensional time-domain BEM analysis, an approximation in time and along the boundary needs to be introduced. This can be done, by using shape functions

$$p(X,t) = \sum_{j=1}^{J}\sum_{m=1}^{M}\phi_p^m(t)\,\eta_p^j(X)\,p_j^m \tag{5}$$

$$\ddot{u}_N(X,t) = \sum_{j=1}^{J}\sum_{m=1}^{M}\phi_{\ddot{u}_N}^m(t)\,\eta_{\ddot{u}_N}^j(X)\,\ddot{u}_{N\,j}^m \tag{6}$$

where the following notation is employed: η_p^j and $\eta_{\ddot{u}_N}^j$ are spatial interpolation functions related to p and $\ddot{u}_N$, respectively, corresponding to a boundary node X_j; ϕ_p^m and $\phi_{\ddot{u}_N}^m$ are time interpolation functions related to p and $\ddot{u}_N$, respectively, corresponding to a discrete time t_m; and finally, $p_j^m = p(X_j,t_m)$ and $\ddot{u}_{N\,j}^m = \ddot{u}_N(X_j,t_m)$.

Taking into account the approximations given in equations (5) and (6), equation (2) can be written at each boundary node; adopting matrix notation, the resulting system for a generic time step $n$is given by

$$\begin{aligned}\left(\mathbf{C}+\mathbf{H}^1\right)\ \mathbf{P}^n = \mathbf{G}^1\ddot{\mathbf{U}}_N^n + \sum_{m=1}^{n-1}\big(\mathbf{G}^{n-m+1}\ddot{\mathbf{U}}_N^m \\ -\mathbf{H}^{n-m+1}\mathbf{P}^m\big)+\mathbf{S}^n\end{aligned} \tag{7}$$

where $\mathbf{H}^n$ and $\mathbf{G}^n$ are the influence matrices computed at the current time step n and $\mathbf{S}^n$ stands for the domain integral indicated in equation (2). After introducing boundary conditions in equation (7), the following expression is obtained

$$\mathbf{A}\,\mathbf{x}^n = \mathbf{B}\,\mathbf{y}^n + \mathbf{Q}^n + \mathbf{S}^n \tag{8}$$

where, as usual in time domain BEM, the entries of $\mathbf{x}^n$ in equation (8) are unknown pressures or normal accelerations at the discrete time t_n, while the entries of vector $\mathbf{y}^n$ are the according known nodal values. $\mathbf{Q}^n$ is the vector related to the convolution process indicated in equation (2); it represents the history up to t_{n-1}. Further details on the implementation of the time-domain BEM algorithm can be found, for instance, in Mansur(1983), Dominguez (1993), Estorff (2000). In order to achieve a more efficient procedure, the convolution process can be properly truncated and the vector $\mathbf{Q}^n$ can be evaluated as it is shown by Soares Jr and Mansur (2004).

Equation (8) yields the dynamic response of the fluid sub-domain at time t_n. In a next step (section 4), it needs to be coupled with the finite element formulation given in section 3.

3 Modelling the FEM sub-domain

The FEM approach adopted in this paper is based on a previous work by Soares Jr. and Mansur (2005). The FEM system of equations, which governs the linear response of a dynamic system, is given by [Hughes (1987), Bathe (1996)]

$$\mathbf{M}\ddot{\mathbf{U}}^n + \mathbf{C}\dot{\mathbf{U}}^n + \mathbf{K}\mathbf{U}^n = \mathbf{R}^n \tag{9}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices respectively; $\mathbf{R}^n$ is the nodal equivalent force vector; $\mathbf{U}^n$, $\dot{\mathbf{U}}^n$ and $\ddot{\mathbf{U}}^n$ are respectively displacement, velocity and acceleration nodal vectors originated from the FEM spatial discretization, at time t_n.

The analytical expressions for the displacement $\mathbf{U}^n$ and the velocity $\dot{\mathbf{U}}^n$ vectors, which obey equation (9), are given by:

$$\begin{aligned}\mathbf{U}^n &= \mathbf{G}^n\,\mathbf{C}\,\mathbf{U}^0 + \dot{\mathbf{G}}^n\,\mathbf{M}\,\mathbf{U}^0 + \mathbf{G}^n\,\mathbf{M}\,\dot{\mathbf{U}}^0 + \mathbf{G}^n \bullet \mathbf{R}^n \\ \dot{\mathbf{U}}^n &= \dot{\mathbf{G}}^n\,\mathbf{C}\,\mathbf{U}^0 + \ddot{\mathbf{G}}^n\,\mathbf{M}\,\mathbf{U}^0 + \dot{\mathbf{G}}^n\,\mathbf{M}\,\dot{\mathbf{U}}^0 + \dot{\mathbf{G}}^n \bullet \mathbf{R}^n\end{aligned} \tag{10}$$

where $\mathbf{G}^n$ represents the Green's function matrix of the model and the symbol $\bullet$ represents convolution. $\mathbf{U}^0$ and $\dot{\mathbf{U}}^0$ are displacement and velocity initial conditions, respectively.

Assuming that a given time-step Δt is small enough, approximation (11) can replace the convolution integrals indicated in equation (10) (f_1 and f_2 are generic functions). It is important to notice that the approximations indicated in equation (11) are analogous to those employed in frequency domain analysis, where standard DFT/FFT algorithms are employed [Soares Jr and Mansur (2003)].

$$\int_0^{\Delta t} f_1(\Delta t-\tau)\, f_2(\tau)\, d\tau = f_1(0)\, f_2(\Delta t)\, \Delta t \tag{11}$$

Taking into account the approximations indicated by equation (11), recursive expressions can be obtained by considering equation (10) at time t_n and by supposing that the analysis starts at time t_{n-1}. The recurrence relations, that arise, are given by:

$$\begin{aligned}\mathbf{U}^n &= (\overline{\mathbf{G}}\mathbf{C}+\dot{\overline{\mathbf{G}}}\mathbf{M})\,\mathbf{U}^{n-1}+\overline{\mathbf{G}}\mathbf{M}\dot{\mathbf{U}}^{n-1}+\mathbf{G}^0\,\mathbf{R}^n\Delta t\\ \dot{\mathbf{U}}^n &= (\dot{\overline{\mathbf{G}}}\mathbf{C}+\ddot{\overline{\mathbf{G}}}\mathbf{M})\,\mathbf{U}^{n-1}+\dot{\overline{\mathbf{G}}}\mathbf{M}\dot{\mathbf{U}}^{n-1}+\dot{\mathbf{G}}^0\,\mathbf{R}^n\Delta t\end{aligned} \tag{12}$$

where $\overline{\mathbf{G}}$ is the Green's function of the model at time step Δt. The $\overline{\mathbf{G}}$ Green's function, as well as its derivatives, can be properly evaluated by solving the system of equations (9) at time $t=\Delta t$, considering a free vibration system submitted to the following initial conditions [Soares Jr and Mansur (2005)]: $\mathbf{G}^0=\mathbf{0}$ and $\dot{\mathbf{G}}^0=\mathbf{M}^{-1}$.

The method considered here for the solution of equations (12) does not use any analytical expression for the problem Green's function; rather it employs the Newmark method to compute numerically the Green's function matrix. The Newmark method expressions are employed initially to establish expressions that permit to compute $\overline{\mathbf{G}}$ and its time derivatives; subsequently the expressions obtained are introduced in the recurrence relations shown by equations (12).

If some simplifications are taken into account, one obtains final recurrence expressions, which permit the establishment of very efficient computational algorithms. These simplifications are, namely: a) consider the damping matrix proportional to the mass matrix ($\mathbf{C}=\alpha_m\mathbf{M}$); b) adopt the trapezoidal rule scheme (γ = 0.50 and β = 0.25) in the Newmark formulation (in fact, it is just necessary to adopt the following relation between the Newmark parameters:$\gamma^2=\beta$, in order to achieve suitable final expressions).

The final recurrence relations that arise, taking into account the above mentioned methodology and simplifications, are given by (for more details see Soares Jr. and Mansur (2005)):

$$\mathbf{U}^n = \overline{\mathbf{U}}+c_3\mathbf{U}^{n-1} \tag{13}$$

$$\dot{\mathbf{U}}^n = c_5\overline{\mathbf{U}}+c_4\mathbf{U}^{n-1}+c_3\dot{\mathbf{U}}^{n-1}+\mathbf{R}^* \tag{14}$$

where

$$\overline{\mathbf{U}} = \mathbf{K}^{*-1}\mathbf{M}\left(c_2\mathbf{U}^{n-1}+c_1\dot{\mathbf{U}}^{n-1}\right) \tag{15}$$

$$\mathbf{K}^* = \mathbf{K}+c_0\mathbf{M} \tag{16}$$

$$\mathbf{R}^* = \mathbf{M}^{-1}\mathbf{R}^n\Delta t \tag{17}$$

and

$$c = (1/(2\beta)-1)\ \alpha_m - 1/(\beta\Delta t) \tag{18}$$

$$c_0 = 1/(\beta\Delta t^2)+(\alpha_m\gamma)/(\beta\Delta t) \tag{19}$$

$$c_1 = (\gamma/\beta-1)\ \alpha_m-(\Delta t/2)(\gamma/\beta-2)\ \alpha_m^2-c \tag{20}$$

$$c_2 = (\alpha_m+c_5)\ c_1 \tag{21}$$

$$c_3 = 1-\Delta t(1-\gamma)\alpha_m+\gamma\Delta t\ c \tag{22}$$

$$c_4 = \alpha_m c_3+c \tag{23}$$

$$c_5 = \gamma/(\beta\Delta t) \tag{24}$$

Once the mass matrix is lumped (in order to avoid solving a system of equations when evaluating the effective force vector **R***, see equation (17)), the above methodology becomes very efficient in a FEM/BEM coupling context, as it will be shown. Equations (13) and (14) enables the computation of the FEM displacement and velocity response, respectively, at time t_n. As it has been shown [Soares Jr and Mansur (2005)], for γ = 0.50 and β = 0.25, the amplification matrix related to the solution algorithm (13)-(14) is second order accurate and unconditionally stable.

4 BEM/FEM coupling procedure

The basic ideas adopted by this work to deal with the coupled analysis can be described as follows: (a) The domain of the original problem is divided into different sub-domains, which are separately modelled by the BEM and the FEM. Structural sub-domains are modelled here with FE while acoustical sub-domains are modelled with BE; (b) The coupling between the BEM and FEM sub-domains is taken into account considering the variables on the sub-domains interfaces, namely: the displacements of the FEM sub-domain interfaces are related with the normal accelerations on the BEM sub-domain interfaces; the pressures on the BEM sub-domain interfaces are related with the nodal forces on the FEM sub-domain interfaces.

The BE formulation adopted by this work, assumes piecewise constant interpolation functions ($\phi^m_{\ddot{u}_N}(t)$) for the normal acceleration (equation (6)). The BEM normal displacement along time ($\mathbf{U}_N^t$), within a time-step ${}_B\Delta t$, can then be obtained by time integration. This procedure gives as result

$$\begin{aligned}\mathbf{U}_N^t &= \mathbf{U}_N^{t_o} + \dot{\mathbf{U}}_N^{t_o}(t-t_o) \\ &+ \ddot{\mathbf{U}}_N^t(t-t_o)^2/2\,,\ \forall t\ \in\ (t_o\,;\ t_o + {}_B\Delta t]\end{aligned} \tag{25}$$

According to equation (25), within a time-step ${}_B\Delta t$, the BEM normal displacement, velocity and acceleration have parabolic, linear and constant behavior along time, respectively (Figure 1(a)). Equation (25) is equivalent to the Newmark method equation for displacement with parameters: $\gamma = 1.00$ and $\beta = 0.50$.

Once equation (25) has been established, one can easily obtain the BEM normal accelerations from the FEM displacements on the interfaces, this procedure is shown on item (3) of the algorithm below. The following algorithm describes the BEM/FEM coupling procedure being adopted (${}_F\Delta t$ and ${}_B\Delta t$ are the FEM and BEM time-steps, respectively, and the initial time attributions are: ${}_Ft = 0$ and ${}_Bt = {}_B\Delta t$):

(1) Begin the evaluations at the current time-step: ${}_Ft = {}_Ft + {}_F\Delta t$; If ${}_F t > {}_Bt$ then: adoption of ${}_Bt = {}_Bt + {}_B\Delta t$ and evaluation of vectors $\mathbf{S}^{Bt}$ and $\mathbf{Q}^{Bt}$;

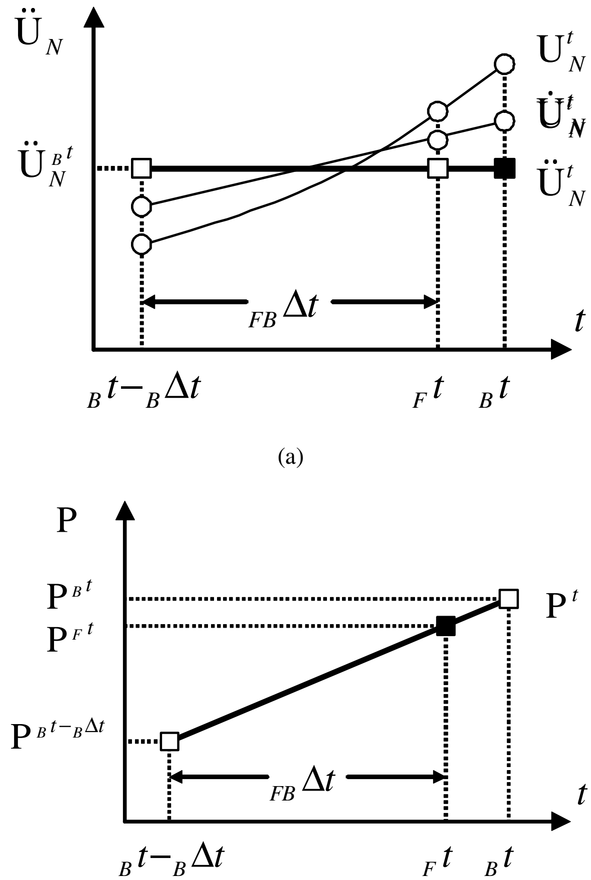
(2) Solve the FEM problem (equation (13)): obtain the displacements on the FEM sub-domain $\mathbf{U}^{Ft}$;

(3) From $\mathbf{U}^{Ft}$ obtain the normal displacement $\mathbf{U}_N^{Ft}$ and the normal acceleration on the BEM interface $\ddot{\mathbf{U}}_N^{Ft} = (2/{}_{FB}\Delta t^2)(\mathbf{U}_N^{Ft} - \mathbf{U}_N^{Bt-B\Delta t}) - (2/{}_{FB}\Delta t)\dot{\mathbf{U}}_N^{Bt-B\Delta t}$ (equation (25));

(4) Time extrapolation of $\ddot{\mathbf{U}}_N^{Ft}$ in order to obtain $\ddot{\mathbf{U}}_N^{Bt}$. Once the interpolation function $\phi_{\ddot{u}_N}^m(t)$ (equation (6)) is usually considered as being piecewise constant, one has: $\ddot{\mathbf{U}}_N^{Bt} = \ddot{\mathbf{U}}_N^{Ft}$ (Figure 1(a));

(5) Solve the BEM problem (equation (8)): obtain the pressure $\mathbf{P}^{Bt}$ on the interface;

(6) Time interpolation of $\mathbf{P}^{Bt}$ in order to obtain $\mathbf{P}^{Ft}$. Once the interpolation function $\phi_p^m(t)$ (equation (5)) is usually considered as being linear, one has: $\mathbf{P}^{Ft} = \mathbf{P}^{Bt}({}_{FB}\Delta t/{}_B\Delta t) + \mathbf{P}^{Bt-B\Delta t}(1 - {}_{FB}\Delta t/{}_B\Delta t)$ (Figure 1(b));



(a)

(b)

Figure 1 : Time interpolation-extrapolation procedures: (a) time extrapolation of $\ddot{\mathbf{U}}_N^{Ft}$ in order to obtain $\ddot{\mathbf{U}}_N^{Bt}$ ($\ddot{\mathbf{U}}_N^{Bt} = \ddot{\mathbf{U}}_N^{Ft}$); (b) time interpolation of $\mathbf{P}^{Bt}$ in order to obtain $\mathbf{P}^{Ft}$ ($\mathbf{P}^{Ft} = \mathbf{P}^{Bt}({}_{FB}\Delta t/{}_B\Delta t) + \mathbf{P}^{Bt-B\Delta t}(1 - {}_{FB}\Delta t/{}_B\Delta t)$).

(7) Considering $\mathbf{P}^{Ft}$ obtain the FEM nodal forces and evaluate the effective load vector $\mathbf{R}^*$ (equation (17));

(8) Solve the FEM problem (equation (14)): obtain the velocities on the FEM sub-domain $\dot{\mathbf{U}}^{Ft}$;

(9) Update results related to the FEM; If ${}_F t + {}_F\Delta t > {}_Bt$ then: update results related to the BEM, including $\mathbf{U}_N^{Bt} = \mathbf{U}_N^{Bt-B\Delta t} + ({}_B\Delta t)\dot{\mathbf{U}}_N^{Bt-B\Delta t} + ({}_B\Delta t^2/2)\ddot{\mathbf{U}}_N^{Bt}$ and $\dot{\mathbf{U}}_N^{Bt} = \dot{\mathbf{U}}_N^{Bt-B\Delta t} + ({}_B\Delta t)\ddot{\mathbf{U}}_N^{Bt}$ (equation (25)).

It is important to notice, that differently from most BEM/FEM acoustic-elastodynamic coupling algorithms, which uses the FEM accelerations for the coupling [Estorff and Antes (1991), Czygan and Estorff (2002), Yu, Lie and Fan (2002), Soares Jr, Estorff and Mansur], the present algorithm works with the FEM displacements. This procedure is generic and can be adopted by other acoustic-elastodynamic coupling methodologies, which do not deal with the structural acceleration on their formulations (BEM/BEM coupling, for instance).

By adopting the FEM time integration methodology here presented, the computation of the displacement at a time t is not dependent of the load at time t. The displacement is in fact dependent of the load history of previous time only (present time excluded) in view of the approximations adopted (equation (11)) and as it is shown by equations (13-14). This fact enables a direct coupling of BEM/FEM procedures by means of the variables on the interface, being an iterative coupling procedure not necessary. Thus, the final coupling algorithm becomes very efficient. Moreover, the FEM time integration methodology here adopted is also theoretically more suitable for BEM/FEM coupling algorithms: once the time integration procedure employed by the BEM is usually based on convolution integrals, it is appropriate to adopt an analogous procedure for the FEM.

In acoustic-elastodynamic interaction analyses, subdomains with completely different properties are usually considered. Thus special procedures to take into account different time discretizations in each sub-domain and to avoid global ill conditioned systems of equations should always be considered.

In order to consider different time discretizations in each sub-domain, special procedures are here adopted, based on interpolations and extrapolations along time, of the variables on the interfaces. These procedures are depicted in Figure 1 and discussed on items (4) and (6) of the proposed algorithm.

The presented algorithm solves BEM and FEM separately, which means that different solution procedures can be applied to solve the BEM and the FEM systems of equations. Thus the symmetry and the sparsity of FEM matrices can easily be taken into account, which results in a more efficient methodology. By solving the BEM and the FEM apart, one also has better conditioned systems of equations, which is important with respect to the accuracy and efficiency of the analysis.

When the FEM time integration methodology presented here is adopted, non-linear analyses can also easily be considered within the FEM sub-domain by means of pseudo-forces [Soares Jr and Mansur (2005)] or any other suitable numerical procedure.

5 Numerical examples

In this section two numerical examples are presented, namely a submerged cylinder subjected to an external explosion and a loaded dam retaining the water of a storage-lake. The results obtained with the BEM/FEM coupling procedure here proposed are compared with other coupling methods, namely, iterative [Soares Jr, Estorff and Mansur] and standard BEM/FEM coupling [Estorff and Antes (1991)].

5.1 Submerged cylinder

This example is concerned with the analysis of an elastic infinite cylinder excited by an acoustic wave caused by an external explosion, as depicted in Figure 2. The properties of the cylinder are: $E = 2.1 \cdot 10^{11} N/m^2$ (Young modulus), $\nu = 0.3$ (Poisson's ratio), $\rho = 7800\ kg/m^3$ (mass density). The properties of the fluid are: $c = 1524\ m/s$ (wave velocity), $\rho = 1000\ kg/m^3$ (mass density). The geometry of the problem is defined by: $r = 0.18m$, $t = 0.0259m$, $d = 1.0m$. The explosion effects are simulated by the following concentrated source: $\widehat{s}(X,t) = s(t)\delta(X-\xi)$, where δ is the Dirac delta function, $\xi = (d,0)$ and $s(t)$ is depicted in Figure 2. 48 linear boundary elements were used to model the fluid and 48 linear quadrilateral finite elements were employed to model the cylinder. The time-step adopted within the BEM sub-domain was ${}_B\Delta t = 0.005ms$; within the FEM sub-domain a time-step ${}_F\Delta t = 0.001ms$ was adopted.

The displacement and hydrodynamic pressure results at points A (α=0) and B ($\alpha = \pi$) are depicted in Figures 3 and 4, respectively. The results obtained by the present work methodology are compared with the results obtained by an iterative BEM/FEM coupling procedure [Soares Jr, Estorff and Mansur]. As it can be seen, the present methodology gives good results without any iterative process for the coupling algorithm; on the other hand, spurious results are achieved with the iterative BEM/FEM coupling procedure if no iterations are considered, as depicted in Figure 3.

Although for usual applications the present methodology is much faster (computational time) than other usual coupling procedures, in the present application this advan-

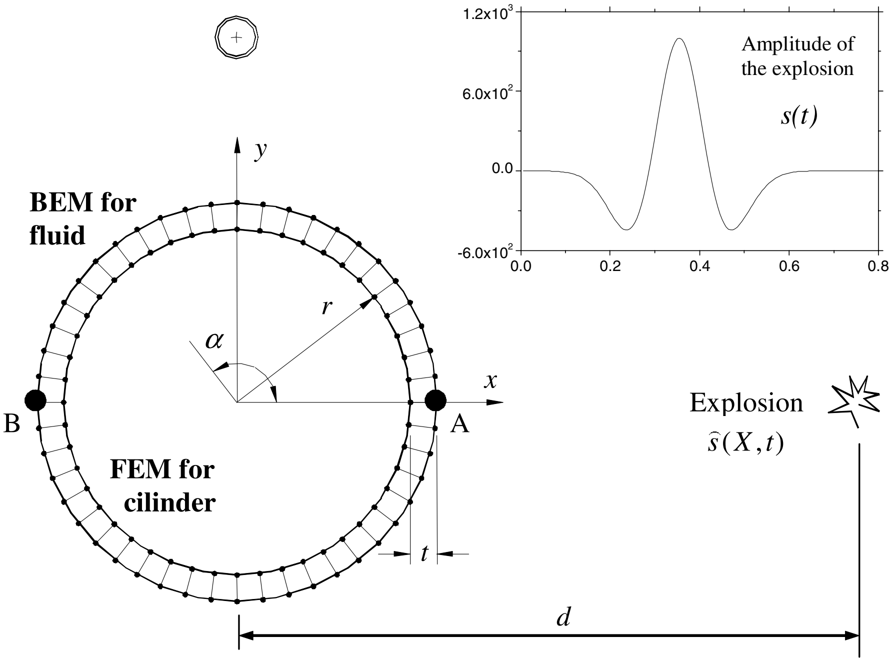


Figure 2 : Submerged cylinder subjected to an external explosion.

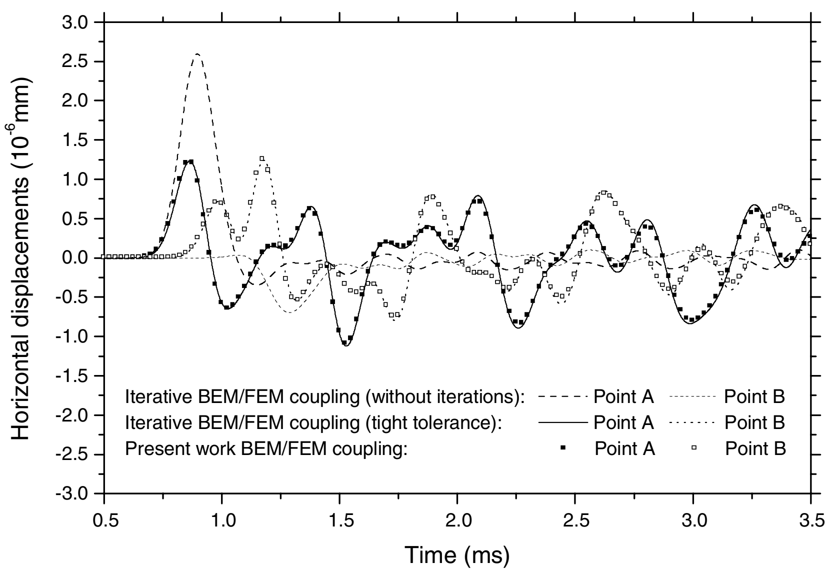


Figure 3 : Horizontal displacements at points A and B of the cylinder surface.

tage is barely observed: most of the computational effort here are due to the BE convolution process, once several time steps are being employed. In the present application, the total CPU time of the proposed methodology is about 90% of the one related to the iterative coupling procedure (an average of 3 iterations per time step was necessary for convergence in the ICA). On the other hand, adopting the convolution truncation process, as proposed by Soares Jr and Mansur (2004), the present total CPU time can be reduced up to 35%, with meaningless loss of accuracy.

5.2 *Storage-lake dam*

In this second example, a dam-reservoir system, as depicted in Figure 5, is analyzed. The structure is subjected to a sinusoidal, distributed vertical load on its crest, acting with an angular frequency $w = 18$ *rad/s*. The material properties of the dam are: Poisson's ratio ν = 0.25; Young's modulus $E = 3.437 \cdot 10^9 N/m^2$; mass density ρ= 2000 kg/m^3. The adjacent water is characterized by a mass density ρ = 1000 kg/m^3 and a wave velocity$c = 1436 m/s$. The time-step adopted for the BEM and FEM are ${}_B\Delta t = 0.00350s$ and ${}_F\Delta t = 0.000875s$, respectively. 93 linear quadrilateral finite elements were employed to model the dam; the number of acoustic boundary elements employed varies with the water level h (see Figure 5), however, in all analyses, linear boundary elements with the same size ($\ell = 5m$) were employed.

The time history of the vertical displacement at point A is shown in Figure 6. Two different water levels, namely $h = 50m$ and$h = 35m$, and their influence on the dam response are investigated. A comparison of the results obtained with the here proposed coupling procedure and with that from the standard coupling scheme used by Estorff and Antes (1991) shows again good agreement. In Figure 7, the transient hydrodynamic pressure at point B is depicted. Also in this case, the results using the standard BEM/FEM coupling procedure match well with those of the new approach (it is important to observe that minor differences between results in Figure 7 are expected once in Estorff and Antes (1991) different time-step and mesh - poorer discretization - were adopted to model the FEM sub-domain).

6 Conclusions

The present paper presented an efficient time-domain BEM/FEM coupling algorithm to solve acoustic-elastodynamic interaction problems. The FEM was employed to model the solid sub-domain (structures or continua) taking into account a time integration scheme based on implicit Green's functions. The classical BEM formulation was adopted to model the acoustic fluid sub-domain. The main advantages of the proposed BEM/FEM coupling procedure are:

- Improved efficiency: (a) lower order systems of equations are considered; (b) different solvers, suitable for each sub-domain, can be employed; (c) no iterative process is necessary in order to take into account the interface coupling conditions.

- Improved accuracy and stability: (a) different time-steps are possible within each sub-domain; (b) the systems of equations of each sub-domain are solved independently, avoiding global ill conditioned systems of equations.

The two applications here presented, comparing the present paper results with those of iterative and standard coupling algorithms, showed the good level of accuracy of the present formulation numerical results (which is also computationally cheaper).

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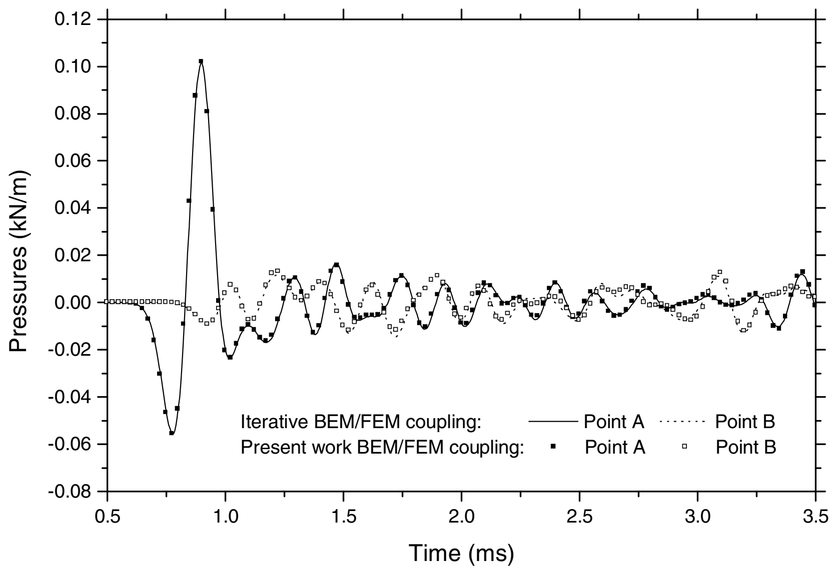

Figure 4 : Hydrodynamic pressures at points A and B of the cylinder surface.

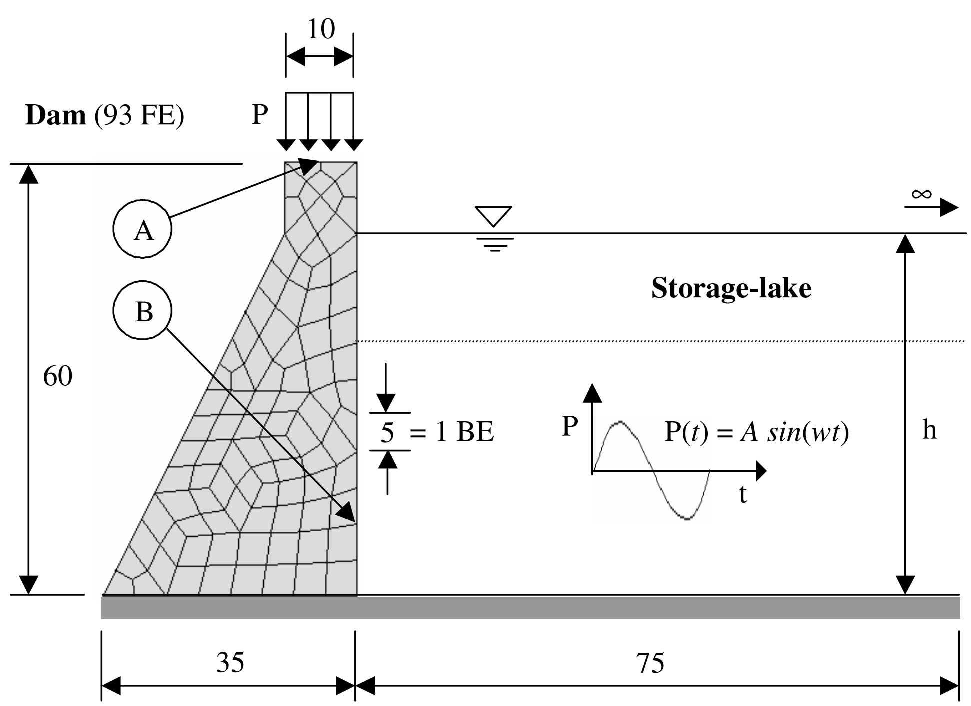

Figure 5 : Dam retaining the water of a semi-infinite storage-lake.

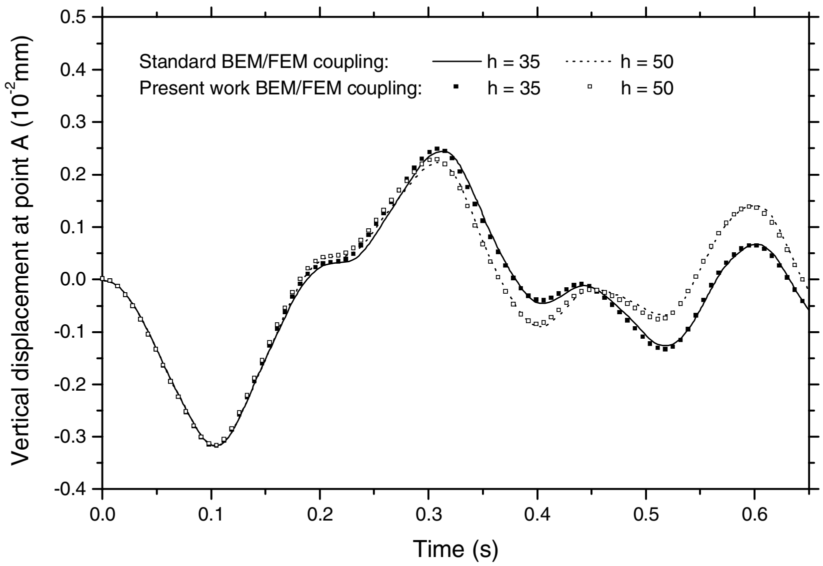

Figure 6 : Vertical displacement at point A: influence of the water level.

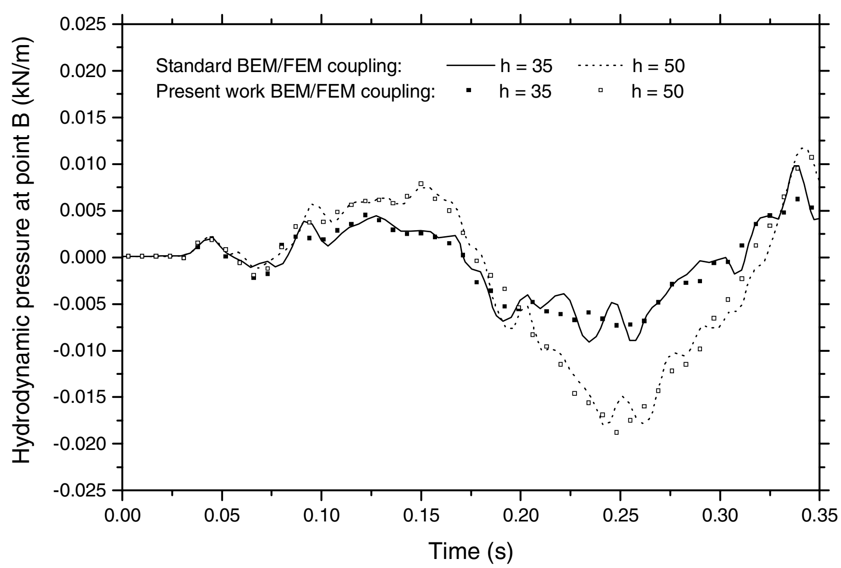

Figure 7 : Hydrodynamic pressure at point B: influence of the water level.

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