Investigation on the Normal Derivative Equation of Helmholtz Integral Equation in Acoustics

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Abstract: Taking the normal derivative of solid angles on the surface into account, a modified Burton and Miller's formulation is derived. From which, a more reasonable expression of the hypersingular operator is obtained. To overcome the hypersingular integral, the regularization scheme developed recently is employed. Plane acoustic wave scattering from a rigid sphere is computed to show the correctness of the modified formulation with the regularization scheme. In the computation, eight-nodded isoparametric element is applied.

keyword: Boundary element method, solid angle, hypersingular integral, Helmholtz integral equation, regularization

1 Introduction

Boundary element method for acoustic problems has been studied for many years. It is well known that the classical boundary element method for exterior acoustic problems fails to provide a unique solution at certain frequencies, which are the characteristics of the associated interior problem. The nonuniqueness is a purely mathematical problem arising from the boundary integral formulation rather than from the nature of the physical problem. Detailed description about the nonuniqueness problem is presented by Schenck (1968), Burton and Miller (1971), Ciskowski and Brebbia (1991), Benthien and Scheck (1996). The composite Helmholtz integral equation (CHIE) presented by Burton and Miller (1971) is one of the most popular approaches to overcome the well-known nonuniqueness problem. It involves a linear combination of the surface Helmholtz integral equation (HIE) and its normal derivative equation. It is a combined use of dual integral representation for acoustic problem

[Chen, Hong (1999)]. Burton and Miller proved that CHIE would yield a unique solution for all frequencies if a suitable complex multiplicative constant was chosen, even if both the HIE and its normal derivative equation suffered from the nonuniqueness problem. This method appears to be more robust for numerical implementation. However, it still suffers from a major drawback that a hypersingular integral is involved in the normal derivative equation of the HIE.

For years, extensive research by Burton and Miller (1971), Meyer, Bell and Zinn (1978), Terai (1980), Mathews (1986), Chien, Rajiyah and Atluri (1990), Wu, Seybert and Wan (1991), Liu and Rizzo (1992), Luke and Martin (1995), Hwang (1997), Wang, Atalla and Nicolas (1997), Yang (1997, 1999), Chen and Liu (1999), Liu and Chen (1999), Gennaretti, Giordani and Morino (1999), Yan (2000), Yan Z.Y., Jiang, He and Yan M. (2001), Liu, Cai, Zhao, Zheng and Lam (2002), Yan, Hung and Zheng (2003) has been devoted to calculate the singular or hypersingular integrals involved in the Burton and Miller (1971) formulation. Based on the different expressions of the hypersingular operator N_d, these methods can be generally divided into two kinds. For one kind, the researchers, Burton and Miller (1971), Meyer, Bell and Zinn (1978), Terai (1980), Mathews (1986), Wu, Seybert and Wan (1991), Liu and Rizzo (1992), Luke and Martin (1995), Wang, Atalla and Nicolas (1997), Chen and Liu (1999), Liu and Chen (1999), Gennaretti, Giordani and Morino (1999), Yan (2000) , Yan Z.Y., Jiang, He and Yan M. (2001), Liu, Cai, Zhao, Zheng and Lam (2002), Yan, Hung and Zheng (2003) took the operator $\partial/\partial n_p$ inside the integral directly. While for the other kind, the researchers, Chien, Rajiyah and Atluri (1990), Hwang (1997), Yang (1997, 1999) assumed that the normal derivative of the solid angles on the surface was zero. Based on such an assumption, they transformed the integral kernel into an integrable form and then took the differential operator into the integral. The numerical results obtained using both of these two different expressions

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agreed well with the corresponding analytical solutions.

Taking the normal derivative of the solid angles on the surface into consideration, Wu (2000) derived a new formulation of the normal derivative of the surface Helmholtz integral equation. He termed it as a global approach for the normal derivative integral equation. No discussion was presented about the expression of the hypersingular operator N_d. Also taking the normal derivative of the solid angles on the surface into account, Liu, Cai, Zhao, Zheng and Lam (2002) developed an improved Burton and Miller's formulation. However, in their derivation, they still expressed the hypersingular operator N_d by taking the operator $\partial/\partial n_p$ inside the integral directly. Therefore, their final equation is not complete.

Recently, several novel boundary integral equations are derived and implemented to avoid the hypersingular integral. Qian, Han, Atluri (2004) [see also Han & Atluri (2003a,b)] derived the symmetric Galerkin boundary element formulations of the regularized forms of non-hypersingular boundary integral equations. Meantime, Qian, Han, Ufimtsev, Atluri (2004) presented the non-hypersingular boundary integral equations by the collocation based boundary element method. Yang (2004) presented the numerical implementation for a boundary integral equation method proposed by Krutitskii (2000).

In this paper, a modified composite Helmholtz integral equation (MCHIE) is presented. The MCHIE takes into account the normal derivative of the solid angles on the surface. We find that the MCHIE is the same as that derived by Chien, Rajiyah and Atluri (1990), Hwang (1997). However, they applied the assumption that the normal derivative of the solid angles on the surface is zero. While in the MCHIE, the normal derivative of solid angles on the surface is taken into consideration. At the same time, a more reasonable explicit expression of the hypersingular operator N_d is derived. The new expression for N_d will overcome the confusion that there are two different expressions for one operator. An improved regularization relationship developed by Yan (2000), Yan Z.Y., Jiang, He and Yan M. (2001), Yan, Hung and Zheng (2003) is successfully applied to overcome the hypersingular integrals in the final equation. Plane acoustic wave scattering from rigid sphere is presented to show the correctness of the MCHIE and the regularization scheme. The numerical results obtained uisng MCHIE agree well with corresponding analytical solutions.

2   Theoretical formulations

Consider the acoustic problem in the exterior domain. The acoustic field is either radiated by a vibrating object or scattered from a rigid structure. Here we assume that the structure has smooth surface. The governing equation for the acoustic pressure $\varphi(p)$ is the Helmholtz integral equation.

$$c(p)\varphi(p) = \iint_S \left[\varphi(q)\frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q)\frac{\partial \varphi(q)}{\partial n_q} \right] dS_q \tag{1}$$

The free-space Green's function G_k for three-dimensional acoustic problem is given by,

$$G_k(p,q) = e^{-ikr}/4\pi r, \qquad r = |p-q| \tag{2}$$

where p and q are respectively the source point and the field point on the structure surface. k is the wave number. $\vec{n}$ represents the inward normal vector, as shown in Fig. 1. $c(p)$ represents the dimensionless solid angle [Ciskowski and Brebbia (1991)] at point p.

$$c(p) = 1 + \iint_S \frac{\partial G_0(p,q)}{\partial n_q} dS_q \tag{3}$$
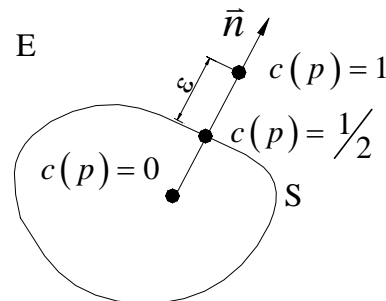
and

$$G_0(p,q) = 1/4\pi r \tag{4}$$

Figure 1 : Dimensionless solid angles along the normal direction

For problems considered currently, we have

$$c(p) = \begin{cases} 1 & p \text{ in the exterior domain} \\ 1/2 & p \text{ on the surfaces} \\ 0 & p \text{ in the interior domain} \end{cases} \tag{5}$$

See Fig. 1, the left and right limits of the normal derivative of the solid angle can be expressed as Eq. (6) and (7). Obviously, the normal derivative of the solid angle does not equal to zero. Strictly speaking, no derivative exits if the function is discontinuous. However for some special functions, general derivatives in some sense exist even if the functions are discontinuous. For example, the Heaviside step function.

$$\lim_{\varepsilon \to 0^-} \frac{\partial c(p)}{\partial n} = \frac{1/2 - 0}{\varepsilon} = \frac{1}{2\varepsilon} \tag{6}$$

$$\lim_{\varepsilon \to 0^+} \frac{\partial c(p)}{\partial n} = \frac{1 - 1/2}{\varepsilon} = \frac{1}{2\varepsilon} \tag{7}$$

In operator notation [Mathews (1986)], Eq. (1) can be written as

$$[-c(p)I + M_k]\varphi = L_k \frac{\partial \varphi}{\partial n} \tag{8}$$

where the integral operators L_k and M_k are defined as

$$L_k \mu = \iint_S \mu(q) G_k(p,q) dS_q \tag{9}$$

$$M_k \mu = \iint_S \mu(q) \frac{\partial G_k(p,q)}{\partial n_q} dS_q \tag{10}$$

It is well known that the classical Helmholtz integral equation Eq. (1) for exterior acoustic problems fails to provide a unique solution at certain characteristic frequencies [Schenck (1968), Burton and Miller (1971), Benthien and Scheck (1996)]. To overcome the nonuniqueness problem, Burton and Miller (1971) developed the composite Helmholtz integral equation (CHIE), which consists of a linear combination of the surface Helmholtz integral equation and its normal derivative equation. In operator notation [Mathews (1986)], it can be expressed as

$$\left\{ -\frac{1}{2}I + M_k + \alpha N_d \right\} \varphi = \left\{ L_k + \alpha \left[\frac{1}{2}I + M_k^T \right] \right\} \frac{\partial \varphi}{\partial n} \tag{11}$$

where α usually takes the value $-i/k$. The integral operators M_k^T and N_d can be expressed as

$$M_k^T \mu = \iint_S \mu(q) \frac{\partial G_k(p,q)}{\partial n_p} dS_q \tag{12}$$

$$N_d \mu = \frac{\partial}{\partial n_p} \iint_S \mu(q) \frac{\partial G_k(p,q)}{\partial n_q} dS_q \tag{13}$$

To derive Eq. (11), Burton and Miller (1971) directly took the normal derivative of the surface Helmholtz equation, which was derived from Eq. (1) as the dimensionless solid angle $c(p)$ equaled to $1/2$. As a result, they had

$$\frac{1}{2} \frac{\partial \varphi(p)}{\partial n_p}$$
$$= \frac{\partial}{\partial n_p} \iint_S \left[\varphi(q) \frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q) \frac{\partial \varphi(q)}{\partial n_q} \right] dS_q \tag{14}$$

In Eq. (1), they applied the condition that the dimensionless solid angles $c(p)$ on surface were constant and equaled to $1/2$. Therefore, they implicitly applied the assumption that on the surface $\partial c(p)/\partial n_p$ equaled to zero. However, Eq. (6) and (7) have shown that $\partial c(p)/\partial n_p$ does not equal to zero even though the dimensionless solid angles $c(p)$ are constant on the surface.

Now, the normal derivative of the dimensionless solid angles will be taken into account to derive the normal derivative equation of the surface Helmholtz integral equation. The derivation is a little different from that presented by Wu (2000).

Taking the normal derivative of Eq. (1) at a point p on the surface,

$$\frac{\partial}{\partial n_p} [c(p)\varphi(p)]$$
$$= \frac{\partial}{\partial n_p} \iint_S \left[\varphi(q) \frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q) \frac{\partial \varphi(q)}{\partial n_q} \right] dS_q \tag{15}$$

Eq. (15) can be rewritten as

$$c(p) \frac{\partial \varphi(p)}{\partial n_p} = -\iint_S \frac{\partial G_k(p,q)}{\partial n_p} \frac{\partial \varphi(q)}{\partial n_q} dS_q$$
$$+ \frac{\partial}{\partial n_p} \iint_S \frac{\partial G_k(p,q)}{\partial n_q} \varphi(q) dS_q - \frac{\partial c(p)}{\partial n_p} \varphi(p) \tag{16}$$

Compared to the corresponding Eq. (1) in the Burton and Miller's formulation (1971), an additional term $-\varphi(p)\partial c(p)/\partial n_p$ appears in the right hand side.

Substituting Eq. (3) into the third term on the right hand

side, Eq. (16) can be rewritten as

$$c(p)\frac{\partial\varphi(p)}{\partial n_p} = -\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_p}\frac{\partial\varphi(q)}{\partial n_q}dS_q$$
$$+\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_q}\varphi(q)\,dS_q$$
$$-\varphi(p)\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q \qquad (17)$$

Then, to ensure the non-integrable singularity in second term on the right hand side of Eq. (17) be integrable, some operation is applied. That is,

$$c(p)\frac{\partial\varphi(p)}{\partial n_p} = -\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_p}\frac{\partial\varphi(q)}{\partial n_q}dS_q$$
$$+\frac{\partial}{\partial n_p}\iint\limits_S \left[\frac{\partial G_k(p,q)}{\partial n_q}\varphi(q) - \frac{\partial G_0(p,q)}{\partial n_q}\varphi(p)\right]dS_q$$
$$+\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}\varphi(p)dS_q$$
$$-\varphi(p)\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q \qquad (18)$$

Therefore

$$c(p)\frac{\partial\varphi(p)}{\partial n_p} = -\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_p}\frac{\partial\varphi(q)}{\partial n_q}dS_q$$
$$+\iint\limits_S \left[\frac{\partial^2 G_k(p,q)}{\partial n_p\partial n_q}\varphi(q) - \frac{\partial^2 G_0(p,q)}{\partial n_p\partial n_q}\varphi(p)\right]dS_q$$
$$-\frac{\partial\varphi(p)}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q$$
$$+\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}\varphi(p)dS_q$$
$$-\varphi(p)\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q \qquad (19)$$

Because

$$\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}\varphi(p)dS_q$$
$$= \varphi(p)\frac{\partial}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q$$
$$+\frac{\partial\varphi(p)}{\partial n_p}\iint\limits_S \frac{\partial G_0(p,q)}{\partial n_q}dS_q \qquad (20)$$

Eq. (19) can be re-written as

$$c(p)\frac{\partial\varphi(p)}{\partial n_p} = -\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_p}\frac{\partial\varphi(q)}{\partial n_q}dS_q$$
$$+\iint\limits_S \left[\frac{\partial^2 G_k(p,q)}{\partial n_p\partial n_q}\varphi(q) - \frac{\partial^2 G_0(p,q)}{\partial n_p\partial n_q}\varphi(p)\right]dS_q \qquad (21)$$

Furthermore,

$$c(p)\frac{\partial\varphi(p)}{\partial n_p} = -\iint\limits_S \frac{\partial G_k(p,q)}{\partial n_p}\frac{\partial\varphi(q)}{\partial n_q}dS_q$$
$$+\iint\limits_S \left[\frac{\partial^2 G_k(p,q)}{\partial n_p\partial n_q} - \frac{\partial^2 G_0(p,q)}{\partial n_p\partial n_q}\right]\varphi(q)\,dS_q$$
$$+\iint\limits_S [\varphi(q) - \varphi(p)]\frac{\partial^2 G_0(p,q)}{\partial n_p\partial n_q}dS_q \qquad (22)$$

Then a modified Burton and Miller's formulation or a modified composite Helmholtz integral equation (MCHIE) is developed by a linear combination of Eq. (22) and Eq. (1) on the surface. In operator notation, it is

$$\left\{-\frac{1}{2}I + M_k + \alpha\left[(N_k - N_0) + N_0 - N_{0c}I\right]\right\}\varphi$$
$$= \left\{L_k + \alpha\left[\frac{1}{2}I + M_k^T\right]\right\}\frac{\partial\varphi}{\partial n} \qquad (23)$$

where N_{0c} is a function of point p rather than an integral operator,

$$N_{0c} = \iint\limits_S \frac{\partial^2 G_0(p,q)}{\partial n_p\partial n_q}dS_q \qquad (24)$$

An improved regularization relationship developed by Yan (2000), Yan Z.Y., Jiang, He and Yan M. (2001), Yan,

Hung and Zheng (2003) can be applied in the numerical computation of Eq. (23). Because Eq. 24 has a hypersingular integral kernel and the integral does not exist in either conventional or Cauchy-principal value sense, it should be obtained in the Hadamard finite part sense [Hwang (1997), Chen J.T., Chen C.T., Chen K.H. and Chen I.L. (2000), Yan, Hung and Zheng (2003), Qian, Han and Atluri (2004)]. Discretized operator matrix D_0 for the hypersingular integral operator N_0 is

$$D_0 = B_0^{-1}(A_0^2 - \frac{1}{4}I) \qquad (25)$$

where

$$B_{0i,m} = \sum_{m=f(j,l)} \iint_{\Delta S_j} N_l G_{0ij}(p,q) dS_q \qquad (26)$$

$$A_{0i,m} = \sum_{m=f(j,l)} \iint_{\Delta S_j} N_l \frac{\partial G_{0ij}}{\partial n} dS_q \qquad (27)$$

While for the discretized matrix D_{0c} about function N_{0c}, it can be found by take the shape functions N_l in Eq. (25) equals to 1. For structures with smooth surfaces, it is easy to obtain that

$$A_{0c} = [c(p) - 1]I = -\frac{1}{2}I \qquad (28)$$

Therefore

$$D_{0c} = B_{0c}^{-1}(A_{0c}^2 - \frac{1}{4}I) = 0 \qquad (29)$$

Eq. (29) turns out that after regularization the term about N_{0c} is a zero matrix.

After regularization, the weakly singular integrals are solved using the scheme proposed by Lachat and Watson (1976).

The original regularization relationship improved by Yan (2000), Yan Z.Y., Jiang, He and Yan M. (2001), Yan, Hung and Zheng (2003) can be derived using the Calderon projector [Chen and Zhou (1992); Chen and Chiu (2002)]. Review of regularization techniques applied to boundary element methods or dual boundary element methods can found in the papers by Tanaka, Sladek V. and Sladek J. (1994), Chen and Hong (1999).

3 Expression of the hypersingular operator N_d

As mentioned in the introduction, in the past decades, a lot of research has been performed on how to calculate the hypersingular integrals involved in the Burton and Miller's formulation (1971). According to the different expressions of the hypersingular operator N_d defined by Eq. (13), these methods can be broadly divided into two kinds. For one kind, the researchers, Burton and Miller (1971), Meyer, Bell and Zinn (1978), Terai (1980), Mathews (1986), Wu, Seybert and Wan (1991), Liu and Rizzo (1992), Luke and Martin (1995), Wang, Atalla and Nicolas (1997), Liu and Chen (1999), Chen and Liu (1999), Gennaretti, Giordani and Morino (1999), Yan (2000), Yan Z.Y., Jiang, He and Yan M. (2001), Liu, Cai, Zhao, Zheng and Lam (2002), Yan, Hung and Zheng (2003) took the operator $\partial/\partial n_p$ inside the integral directly and they had

$$N_d \varphi = \iint_S \frac{\partial^2 G_k(p,q)}{\partial n_p \partial n_q} \varphi(q) dS_q \qquad (30)$$

While for the other kind, the researchers, Chien, Rajiyah and Atluri (1990), Hwang (1997), Yang (1997, 1999) assumed that the normal derivative of the solid angles on the surface was zero. Based on such an assumption, they transformed the integral kernel into an integrable form and then took the differential operator into the integral. Finally, they had

$$N_d \varphi = \iint_S \left(\frac{\partial^2 G_k(p,q)}{\partial n_p \partial n_q} \varphi(q) - \frac{\partial^2 G_k(p,q)}{\partial n_p \partial n_q} \varphi(p) \right) dS_q \qquad (31)$$

The numerical results obtained using both of these two different expressions agreed well with the corresponding analytical solutions. Then we want to know which one is more reasonable.

From Eq. (16) and (21), we have

$$\frac{\partial}{\partial n_p} \iint_S \frac{\partial G_k(p,q)}{\partial n_q} \varphi(q) dS_q = \frac{\partial c(p)}{\partial n_p} \varphi(p)$$

$$+ \iint_S \left[\frac{\partial^2 G_k(p,q)}{\partial n_p \partial n_q} \varphi(q) - \frac{\partial^2 G_0(p,q)}{\partial n_p \partial n_q} \varphi(p) \right] dS_q \qquad (32)$$

That is

$$N_d \varphi = \frac{\partial c(p)}{\partial n_p} \varphi(p)$$

$$+ \iint_S \left[\frac{\partial^2 G_k(p,q)}{\partial n_p \partial n_q} \varphi(q) - \frac{\partial^2 G_0(p,q)}{\partial n_p \partial n_q} \varphi(p) \right] dS_q \quad (33)$$

Even though the results obtained using Eq. (30) and (6) are the same, we must point out that both Eq. (30) and (6) are not very strict expressions for operator N_d. The MCHIE method developed here and Eq. (33) may help the researchers to get a better understanding on the well-known hypersingular integral operator N_d.

4   Numerical examples

Plane acoustic wave $\varphi_I = \varphi_0 e^{-ikz}$ scattering from a rigid sphere of radius a is calculated to show the correctness and feasibility of the modified Burton and Miller's formulation with the improved regularization scheme. Eight-nodded isoparametric element is applied in the computation. Some commercial FEM-codes are available to generate meshes as shown in Fig. 2. In this study, the 832-element model is generated using ANSYS with the element type SHELL93. This example is computed using the in-house developed code, SSFI (Sound Structure Fluid and their Interaction).

For plane acoustic wave scattering from a rigid sphere, the analytical solution [Junger (1952)] of the scattered acoustic pressure $\varphi_s(r, \theta)$ at a distance r from the center of the sphere and at an angle θ from the direction of the incoming wave is given by

$$\frac{\varphi_s(r,\theta)}{\varphi_0}$$

$$= \sum_{m=0}^{\infty} \left[-(-i)^m (2m+1) \frac{j'_m(ka)}{h'_m(ka)} \right] h_m(kr) P_m(\cos\theta) \quad (34)$$

where j_m is spherical Bessel function of the first kind and h_m is spherical Hankel function of the second kind. P_m denotes Legendre polynomial of order m.

The surface Helmholtz integral equation (HIE) for the scattering of a plane acoustic wave from a rigid sphere is given by

$$\left[-\frac{1}{2} I + M_k \right] \varphi = \varphi_I \quad (35)$$
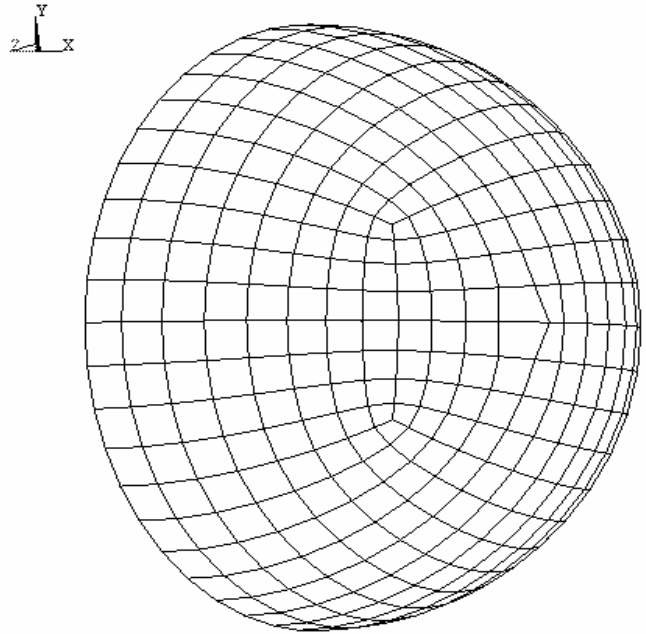

Figure 2 : Discretization for half of a sphere surface with 416 elements

The modified composite Helmholtz integral equation (MCHIE) for the scattering of a plane acoustic wave from a rigid sphere is given by

$$\left[-\frac{1}{2}I + M_k + \alpha[(N_k - N_0) + N_0 - N_{0c}I] \right] \varphi$$

$$= \varphi_I + \alpha \frac{\partial \varphi_I}{\partial n} \quad (36)$$

where the acoustic pressure $\varphi = \varphi_s + \varphi_I$.

Dimensionless scattered surface acoustic pressures at points $\theta = 0$ and $\theta = \pi$ are presented as function of the reduced wave number ka in Fig. 3 and 4. Numerical results obtained using HIE and MCHIE are compared with the corresponding analytical solutions. It is clear that at certain characteristic frequencies the HIE can not provide unique solutions. More numerical examples to show the nonuniqueness problem can be found in the papers by Chen J.T., Chen C.T., Chen K.H. and Chen I.L. (2000), Yan, Hung and Zheng (2003), Qian, Han and Atluri (2004), Qian, Han, Ufimtsev and Atluri (2004). To show the directivity of the scattered acoustic field, dimensionless scattered acoustic pressures on the surface of the sphere are displayed in Fig. 5 to 8 as reduced wave number $ka = 10$, 4π, 5π and 6π. Numerical results obtained
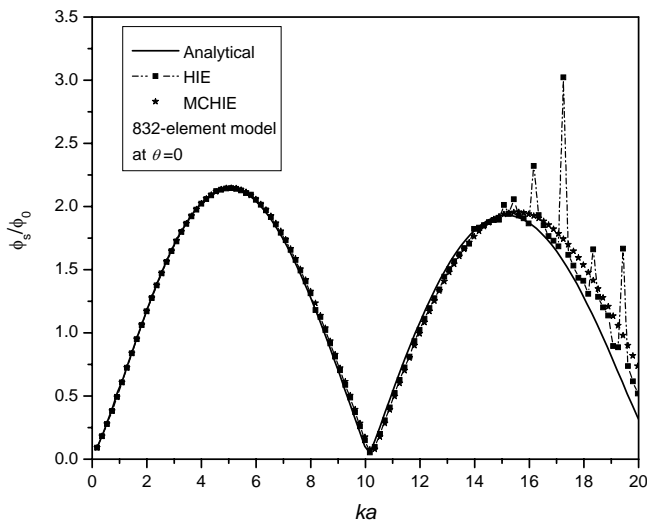
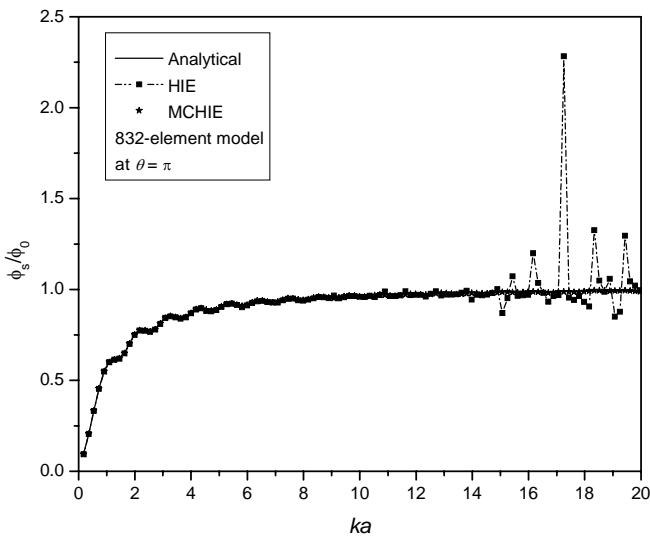**Figure 3** : Dimensionless scattered acoustic pressure at $\theta = 0$ as a function of ka

Figure 4 : Dimensionless scattered acoustic pressure at $\theta = \pi$ as a function of ka

Figure 5 : The angular dependence of scattered acoustic as $ka = 10$
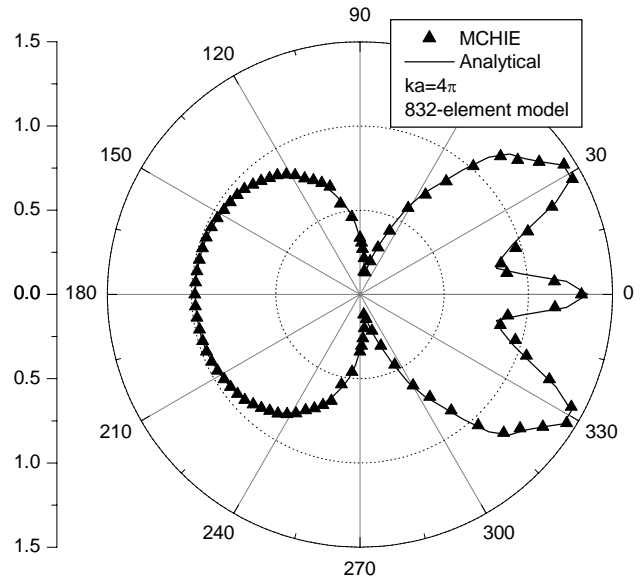
Figure 6 : The angular dependence of scattered acoustic pressures as $ka = 4\pi$

using MCHIE are compared with the corresponding analytical solutions. Since $ka = 10$ is not a characteristic frequency, the numerical results obtained using HIE also displayed in Fig. 5 for comparison with that obtained using MCHIE and analytical solutions. Clearly, all the numerical results obtained using MCHIE agree well with the corresponding analytical solutions.
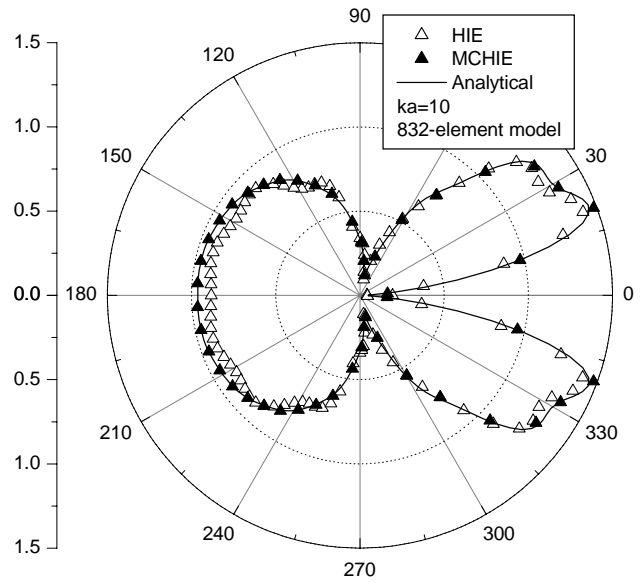
5　Conclusion

In this paper, taking the normal derivative of the solid angles on the surface into consideration, a modified Burton and Miller's formulation (MCHIE) is derived. From which, a more reasonable expression of the hypersingu-
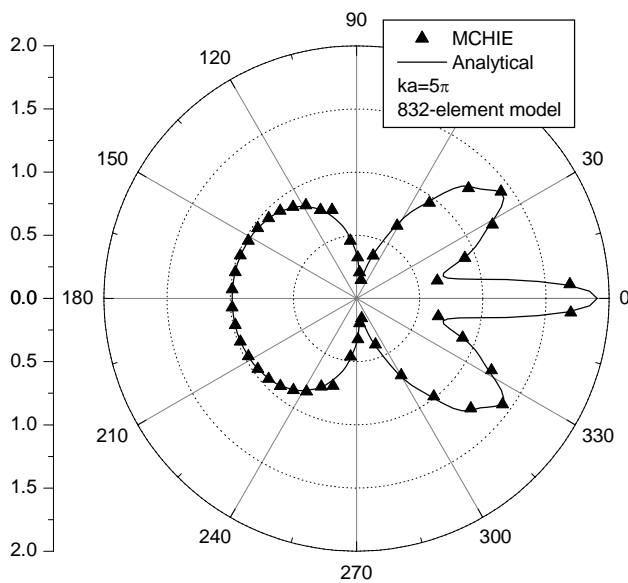
Figure 7 : The angular dependence of scattered acoustic pressures as $ka = 5\pi$
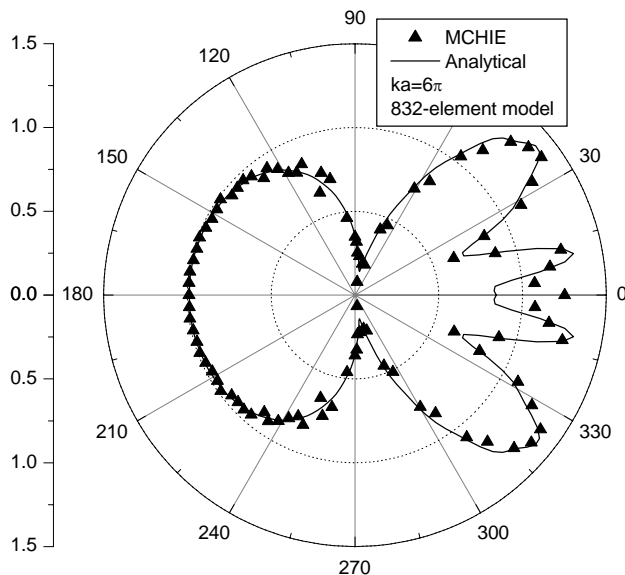

Figure 8 : The angular dependence of scattered acoustic pressures as $ka = 6\pi$

lar operator N_d is derived. This expression will help researchers to avoid the confusion from different expressions for the same integral operator. An improved regularization formulation is successfully employed to overcome the hypersingular integrals. Plane acoustic wave scattering from a rigid sphere is computed to validate the MCHIE with the improved regularization scheme. The numerical results obtained using MCHIE agree well with the corresponding analytical solutions.

References

Benthien, W.; Schenck, A. (1996): Nonexistence and nonuniqueness problems associated with integral equation methods in acoustics. *Computers & Structures* 65: 295-305.

Burton, A. J.; Miller, G. F. (1971): The application of integral equation methods to the numerical solution of some exterior boundary value problems. *Proc.R.Soc.London Ser.* A323: 201-210.

Chen, J. T.; Chen, C. T.; Chen, K. H.; Chen, I. L. (2000): On fictitious frequencies using dual BEM for non-uniform radiation problems of a cylinder. *Mechanics Research Communications* 27(6): 685-690.

Chen, J. T.; Chiu, Y. P. (2002): On the pseudo-differential operators in the dual boundary integral equations using degenerate kernels and circulants. *Engineering Analysis with Boundary Elements* 26(1): 41-53.

Chen, J. T.; Hong, H. K. (1999): Review of dual boundary element methods with emphasis on hypersingular integrals and divergent series. *ASME Appl. Mech. Rev.* 52(1): 17-33.

Chen, S. H.; Liu, Y. J. (1999): A unified boundary element method for the analysis of sound and shell-like structure interactions. I Formulation and Verification. *J. Acoust. Soc. Am.* 106: 1247-1254.

Chen, G.; Zhou, J. (1992): Boundary element method. Academic Press: 93-105.

Chien, C. C.; Rajiyah, H.; Atluri, S. N. (1990): An effective method for solving the hypersingular integral equations in 3-d acoustics. *J. Acoust. Soc. Am.* 88: 918-937.

Ciskowski, R. D.; Brebbia, C. A. (1991): Boundary element methods in acoustics. Computational Mechanics Publications, Souththampton Boston: 65-75.

Gennaretti, M.; Giordani, A.; Morino, L. (1999): A third-order boundary element method for exterior acoustics with application to scattering by rigid and elastic shells. *J. SounVib.* 222: 699-722.

Han, Z. D.; Atluri, S. N. (2003a): On Simple Formulations of Weakly-Singular Traction & Displacement

BIE, and Their Solutions through Petrov-Galerkin Approaches, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 4, No. 1, pp. 5-20

Han, Z. D.; Atluri, S. N. (2003b): Truly Meshless Local Petrov-Galerkin (MLPG) Solutions of Traction & Displacement BIEs, *CMES:Computer Modeling in Engineering & Sciences*, Vol. 4, No. 6, pp. 665-678

Hwang, W. S. (1997): Hypersingular boundary integral equations for exterior acoustic problems. *J. Acoust. Soc. Am.* 101: 3336-3342.

Junger, M. C. (1952): Sound scattering by thin elastic shells. *J. Acoust. Soc. Am.* 24: 366-373.

Krutitskii, P. A. (2000): A new approach to reduction of the Neumann problem in acoustic scattering to a non-hypersingular integral equation. *IMA J. Appl. Math.* 64: 259-269.

Lachat, J. C.; Watson, J. O. (1976): Effective numerical treatment of boundary integral equations. *Int. J. Num. Methods Eng.* 10: 991-1005.

Liu, G. R.; Cai, C.; Zhao, J.; Zheng, H.; Lam, K. Y. (2002): A study on avoiding hypersingular integrals in exterior acoustic radiation analysis. *Applied Acoustics* 63: 643-657

Liu, Y. J.; Chen, S. H. (1999): A new form of the hypersingular boundary integral equation for 3-D acoustics and its implementation with C^0 boundary elements. *Comput. Methods Appl. Mech. Eng.* 173: 375-386.

Liu, Y. J.; Rizzo, F. J. (1992): A weakly-singular form of the hypersingular boundary integral equation applied to 3-D acoustic wave problems. *Comput. Methods Appl. Mech. Eng.* 96: 271-287.

Luke, C. J.; Martin, P. A. (1995): Fluid-solid interaction: acoustic scattering by a smooth elastic obstacle. *SIAM J.Appl.Math.* 55: 904-922.

Mathews, I. C. (1986): Numerical techniques for three dimensional steady-state fluid-structure interaction. *J. Acoust. Soc. Am.* 79: 1317-1325.

Meyer W. L.; Bell, W. A.; Zinn, B. T. (1978): Boundary integral solutions of three dimensional acoustic radiation problems. *J. Sound Vib*. 59: 245-262.

Qian, Z. Y.; Han, Z. D.; Atluri, S. N. (2004): Directly derived non-hypersingular boundary integral equations for acoustic problems, and their solution through Petrov-Galerkin schemes. *CMES: Computer Modeling in Engineering & Sciences* 5(6): 541-562.

Qian, Z. Y.; Han, Z. D.; Atluri, S. N. (2004): Erratum: "Directly derived non-hypersingular boundary integral equations for acoustic problems, and their solution through Petrov-Galerkin schemes" *CMES: Computer Modeling in Engineering & Sciences,* vol 5, no. 6, pp 541-562, 2004). *CMES: Computer Modeling in Engineering & Sciences* 6 (1): 115-122.

Qian, Z. Y.; Han, Z. D.; Ufimtsev, P.; Atluri, S. N. (2004): Non-hypersingular boundary integral equations for acoustic problems, implemented by the collocation-based boundary element method. *CMES: Computer Modeling in Engineering & Sciences* 6(2): 133-144.

Schenck, H. A. (1968): Improved integral formulation for acoustic radiation problems. *J. Acoust. Soc. Am.* 44: 41-58.

Tanaka, M.; Sladek, V.; Sladek, J. (1994): Regularization techniques applied to boundary element methods. *ASME Appl. Mech. Rev.* 47(10): 457-499.

Terai, T. (1980): On calculation of sound fields around three dimensional objects by integral equation methods. *J. Sound Vib.* 69: 71-100.

Wang, W. P.; Atalla, N.; Nicolas, J. (1997): A boundary integral approach for acoustic radiation of axisymmetric bodies with arbitrary boundary conditions valid for all wave numbers. *J. Acoust. Soc. Am.* 101: 1468-1478.

Wu, T. W. (2000): Boundary Element Acoustics: Fundamentals and Computer Codes. WIT, Southampton: 69-74.

Wu, T. W.; Seybert, A. F.; Wan, G. C. (1991): On the numerical implementation of a Cauchy principal value integral to insure a unique solution for acoustic radiation and scattering. *J. Acoust. Soc. Am.* 90: 554-560.

Yan, Z. Y. (2000): Sound transmission through thin elastic shell with different fluids on both the inside and outside. Ph. D Thesis, Shanghai Jiao Tong University, P.R. China.

Yan, Z. Y.; Hung, K. C.; Zheng, H. (2003): Solving the hypersingular boundary integral equation in three-dimensional acoustics using a regularization relationship. *J. Acoust. Soc. Am.* 113: 2674-2683.

Yan, Z. Y.; Jiang, J.; He, Y. S.; Yan, M. (2001): New approach used to deal with hypersingular numerical integral in acoustic boundary element method. *Shengxue Xuebao/Acta Acustica* (in Chinese) 26: 282-286.

Yang, S. A. (1997): Acoustic scattering by a hard

and soft body across a wide frequency range by the Helmholtz integral equation method. *J. Acoust. Soc. Am.* 102: 2511-2520.

Yang, S. A. (1999): A boundary integral equation method for two-dimensional acoustic scattering problems. *J. Acoust. Soc. Am.* 105: 93-105.

Yang, S. A. (2004): An integral equation approach to three-dimensional acoustic radiation and scattering problems. *J. Acoust. Soc. Am.* 116(3): 1372-1380.