The Characteristic Basis Function Method: A New Technique for Fast Solution of Radar Scattering Problems

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Abstract: In this paper, we introduce a novel approach for the efficient solution of electromagnetic scattering problems from objects that can be represented in terms of facets. The approach is based on the use of the Characteristic Basis Functions (CBFs), which are high-level basis functions of special types, and whose domains are not bound by the conventional Rao, Wilton and Glisson (RWG) discretization using triangular patches that are typically $\lambda/10$ to $\lambda/20$ in size. In contrast, the CBFs are defined over much larger-size domains, even tens of wavelengths in size, with no limit placed on the dimensions of the facets. The use of these basis functions leads to relatively small-size matrices, typically orders of magnitude smaller than the conventional ones generated by the RWG bases; and yet, the reduced matrices are sparse and well-conditioned in nature, which is typically not the case when conventional entire domain basis functions are used instead. A novel technique for the construction of CBFs, which is based on the Windowed Plane Wave Spectrum (WPWS) approach that totally bypasses the RWG discretization, associated matrix generation, or its solution, is presented in the paper. Some representative examples that illustrate the accuracy of the CBF approach are included, and the numerical efficiency of the CBF approach over the conventional integral equation formulation and matrix solution, including the Fast Multipole Method (FMM), is demonstrated for a class of problems whose geometries can be represented in terms of facets.

keyword: Characteristic basis functions, Method of Moments, Electromagnetic Scattering, Radar Cross Section (RCS).

1 Introduction

Conventional approaches to solving a scattering problem by using the Method of Moments (MoM) involve the discretization of the object geometry as a first step. It is a common practice to use the Rao, Wilton and Glisson (RWG) basis functions [Rao, Wilton, and Glisson (1982)] in the MoM formulation, with a triangular patch discretization whose size ranges from $\lambda/10$ to $\lambda/20$. Thus, as the object dimensions become large in terms of the wavelength, the size of the associated MoM matrix grows very rapidly, and this, in turn, places an inordinately heavy burden on the CPU in terms of both memory and time. To-date, attempts to circumvent these problems by using entire domain basis functions have only been marginally successful, because these functions tend to make the resulting matrix highly ill-conditioned. Hybridization of the MoM with asymptotic techniques, such as the Physical Optics or ray-based methods, has been proposed by a number of authors [Hodges and Rahmat-Samii (1994,1997); Zhongxiang and Volakis (1999)] as alternatives to the use of conventional entire domain basis functions, though this strategy has only met with a limited success owing to a lack of methodologies that provide a seamless merger of the numerically rigorous and asymptotic techniques.

Recently, the Fast Multipole Method (FMM) has been introduced to circumvent some of the problems alluded to above [Engheta, Murphy, Rokhlin, and Vassiliou (1992); Song, Lu, Chew (1997); Geng, Sullivan, and Carin (2000); and more recently Chew et al. (2003)]. The FMM has enabled us to take a quantum leap into the realm of CEM capabilities. The FMM and its multi-level versions have made it possible to analyze scatter-ers *via* the MoM technique that are orders of magnitude larger in size than could be handled just a few years ago. The FMM realizes a saving in the memory requirements by storing only the near-field interaction part of the full matrix, and by carrying out the matrix vector product needed in the iterative solvers—that are almost always employed for the solution of large matrices—in a highly efficient manner using the spherical harmonic expansion technique. But even the FMM is locked into a geome-

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try discretization size ranging from λ/10 to λ/20, which makes the near-field interaction part of the MoM matrix still grow at a rapid pace as the object becomes electrically large.

The macro basis function (MBF) approach and a simple recursive technique called a subdomain multilevel approach (SMA) have been recently developed to handle large planar antenna arrays [Suter and Mosig (2000); Hurst (2000)], and a similar technique for microstrip circuit analysis (including arrays) has been proposed by Matekovits, Vecchi, Dassano, and Orefice (2001). These approaches entail the analysis of partial domains of the original geometry for the construction of the MBFs. The domains of the MBFs are larger than that used in the conventional MoM formulation using the RWGs; hence, this leads to a reduction in the number of unknowns. While the MBF approach has been shown to be often quite accurate for planar antenna arrays, at least away from the frequencies where the inter-element coupling is negligible, it is not obvious how we might generalize it to handle arbitrary three dimensioned objects that are normally encountered in EM scattering problems of the type investigated in this paper. Furthermore, it may not be evident to the user how the MBF approach can provide an independent indication of the convergence of the solution, and what might be a systematic approach to improving the solution, if such an improvement is warranted.

The technique introduced in this paper differs from the MBF and related approaches, significantly, in several aspects. First, the technique presented herein is more general, and can be applied to any arbitrary, three-dimensional, faceted surfaces. Second, it includes the mutual coupling effects rigorously, and yet reduces the number of degrees of freedom (DoFs) dramatically in comparison to the conventional RWG formulation. Third, it uses a new type of high level basis functions, referred to herein as primary and secondary Characteristic Basis Functions (CBFs), which are used to represent the unknown induced current. Unlike the RWG bases, the CBFs are specially constructed to fit the problem geometry, and this is done by incorporating the physics of the problem into the basis functions. The use of the CBFs not only assures that the solution will naturally tend to the asymptotic limit, it also obviates the need to hybridize them with other basis functions derived by using asymptotic methods, *e.g.*, GTD or PO/PTD. Fourth, and perhaps one of the most unique features of these ba-

sis functions is that for objects represented by facet-type of discretization as shown in Fig.1 (facet sizes can be *arbitrarily large* in terms of the wavelength), the above basis functions can be generated without solving an integral equation and, hence, the formulation totally bypasses the usual RWG discretization and associated matrix generation or solution. Although in the asymptotic limit the dominant term of the CBF is the Physical Optics (PO) current, it uses several hundred (or even thousands) additional terms to account for the contributions of the other facets. Thus, it is able to capture of the nuances of the shadow and transition regions and the traveling wave contributions that are difficult to add-on to the GTD and PO/PTD algorithms for complex structures, for little additional cost. To the best of the knowledge of the authors, this novel feature is not available in any other approaches to constructing the high-level basis functions. We should mention that the CBFs were originally introduced for the analysis of microstrip circuits and antennas by Mittra, Du, Prakash, Yeo, and Kwon (2002), but the technique for constructing them for faceted objects by using a matrix-free approach, which is described in this work, was not mentioned by them, because the matrix-free approach is not applicable to microstrip circuit problems. Finally, we mention that the concept of CBF is also very different from the higher-order basis function employed in the time domain by Hesthaven et al. (2003).
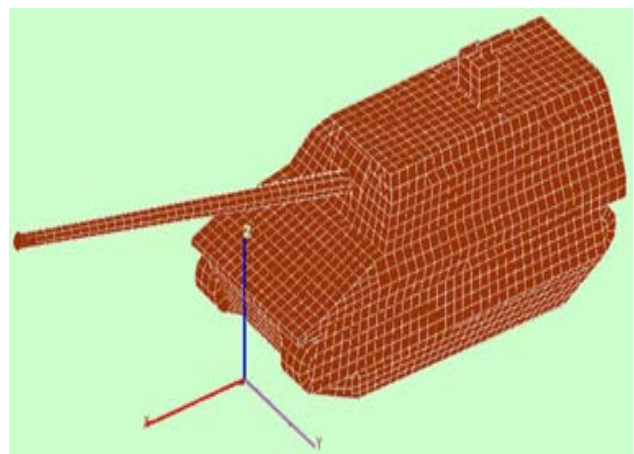

Figure 1 : A faceted geometry.

To demonstrate the effectiveness of the CBFs, we present numerical solutions for the problem of scattering from a

class of geometries that serve to illustrate the applicability, accuracy, and numerical efficiency of these new basis functions. The results obtained via the use of the CBFs are compared with those derived by using the conventional triangular patch discretization with RWG basis functions to illustrate the significant computational advantages gained by using the CBFs is evident from these examples.

2 Construction of Primary Characteristic Basis Functions

We now outline the procedure for constructing the primary basis functions [see Prakash and Mittra (2003)] for a planar facet with a polygonal boundary. The matrix-free approach to constructing the CBF for this geometry is based on a windowed plane wave spectrum (WPWS) approach by Mittra, and Prakash (2002) for truncated periodic structures, *e.g.*, Frequency Selective Surface (FSS) radomes, and later extended to the problem of scattering from a plate by Monorchio, Tiberi, Manara, and Mittra (2002). However, our approach for constructing the CBFs is different from their work because we are assuming that the facet #m does not have a free edge, *i.e.*, it is connected to other facets surrounding it (see Fig. 2a). An important consequence of dealing with this type of facet is that the CBFs we generate will not have the singularity in the induced currents associated with the free edges. On the other hand, we can readily extend the procedure (and in fact we already have) to the cases where one or more edges of the facet are free, by including the requisite singular behavior in the basis functions.

The steps for WSPS approach are given below:

- Step-1. Window the incident plane wave such that is non-zero only on the extended facet # m (see Fig.2b), which is realized by adding a $\lambda/5$ wide border to it.

- Step-2. Extend the facet to an infinite plane and solve for the induced current on that plane for each of the constituent plane waves of the windowed incident field. Note that the implementation of this step is trivial, and requires no matrix generation, or solution. In addition, the time required to construct the basis functions is fast (see examples given in sec. 4), even though we may include several hundred constituent homogeneous and inhomogeneous

(visible and invisible) plane waves to represent the windowed plane waves incident on the facet.

- Step-3: Retain only the portion of the current on the original facet and define it as the primary characteristic basis function
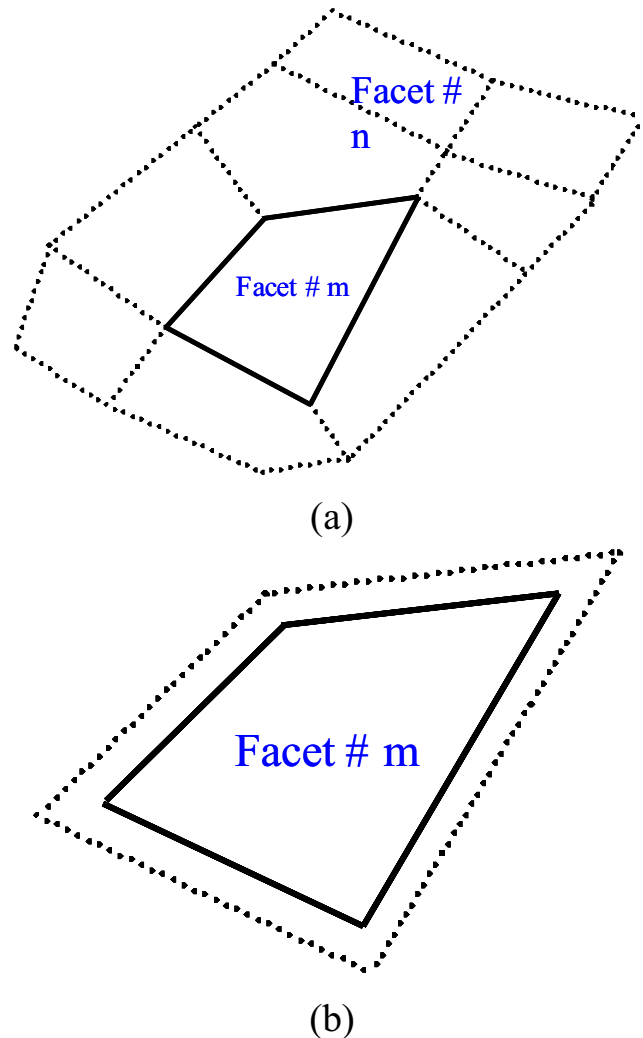

(a)

(b)

Figure 2 : (a) Facet # m illuminated by a plane wave. It is assumed that the facet has no free edges; (b) extended facet # m.

3 Generation of Secondary Basis Functions

The secondary basis functions are the currents induced on different facets, say facet # n in Fig. 2a for example, by the primary CBF residing on facet # m. The two

facets need not be co-planar and their orientations may be arbitrary.

The form of the CBF generated by the technique described in the last paragraph is such that their plane wave spectra are readily obtainable. We take advantage of this fact in computing the secondary basis functions, and follow the same procedure as that employed for the primary CBFs, except that we now have a spectrum of plane waves incident on the facet # n rather than a single one as before when the primary ones were derived.

This unique feature of the CBFs enables us, once again, to construct the secondary ones without the use of RWGs, matrix generation or solution. As pointed out before, this feature for constructing the CBFs is not found in other approaches to generating high-level basis functions. Of course, one can always exercise the option of using the RWG formulation of the original problem as a starting point, and construct the CBFs by solving blocks of matrices that are much smaller than the original one, albeit at an increased cost in the computation time. In fact, in some cases it may well be necessary to follow this approach when a facet in question has fine features, *e.g.*, has a thin wire antenna attached to it, or has a conformal antenna embedded in it. What is important, however, that regardless of how the CBFs are generated, their use results in a reduced matrix, whose derivation is described below.

4 Reduced Matrix Generation and Solution

The third step in the solution process is to generate a reduced matrix by using the Galerkin procedure. A typical element of the reduced matrix may be expressed as $<E_m, J_n>$, where E_m is the tangential electric field induced on the facet # n by the primary (or secondary) basis function residing on the facet # m (see Fig. 2); and, J_n is the testing function on the facet # n, which is also one of the primary (or secondary) basis functions, previously generated by using the procedure described in the last section.

The reduced matrix, which is orders of magnitude smaller in size than its RWG counterpart, can now be solved directly, without the use of iteration in many practical cases, and the RCS can then be computed in the usual way. We have found that the reduced matrices generated by using the CBF's are not only well-conditioned, but quite sparse as well. We conjecture, therefore, that sparse matrix solvers can be successfully used to carry out a direct LU decomposition of the reduced matrix when it is large, and one may seldom need to resort to iterative solvers for a wide range of problems formulated by using the CBFs. Some examples of the reduced matrix sizes are given in the next section where the corresponding size of the conventional RWG matrix is also given for comparison.

5 Illustrative Numerical Results

To illustrate the accuracy of the procedure we compare the field radiated by the CBF generated for a 3.8λ x 3.8λ plate (the extended plate size is 4λ x 4λ), and compare this radiated field with that generated by using approximately 5000 RWG basis functions. The comparison, shown in Fig.3, is seen to be very good. (Through not shown here, the induced currents also compare very well with each other). It has also been verified that the boundary condition on the tangential E-field is indeed well satisfied on the PEC plate by the scattered field generated by the CBF.
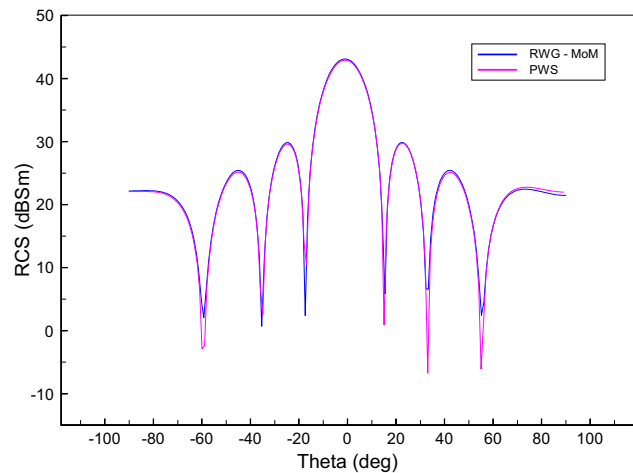

Figure 3 : Field radiated by the CBF of a 3.8λ x 3/8λ PEC plate at $\theta_i = \phi_i = 1°$, TE pol.

To demonstrate the power of this approach we repeat the same procedure for a 9.8λ x 9.8λ plate (extended size 10λ x 10λ), illuminated by a plane wave incident at an angle of θ=30°. Figure 4 shows the comparison of the fields radiated by the RWG currents, and by the CBF (*one single basis function for the entire plate*) derived by using the matrix-free method described above. The time taken to generate the CBF is still <10 secs, (and it
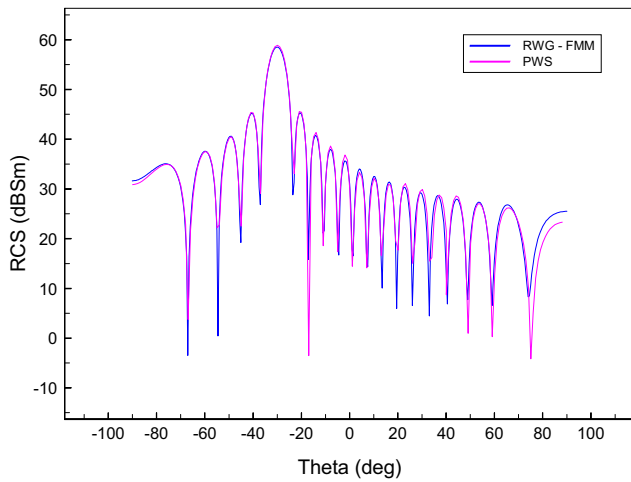
Figure 4 : Field radiated by the CBF of a 9.8λ x 9/8λ PEC plate at $\theta_i = 30°, \phi_i = 1°$, TE pol.

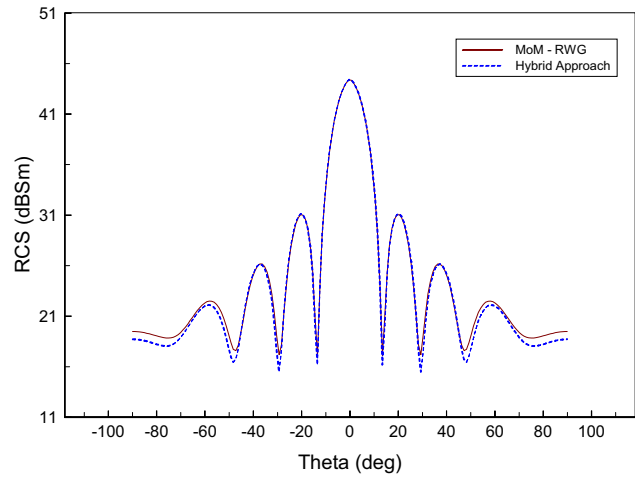**Figure 5b** : RCS of the 4λ x 4λ PEC plate computed by using PWS and RWG basis.

is virtually independent of the size of the facet), whereas the a multilevel FMM code to generate the same CBF requires 29,800 unknowns, and approximately 30 min. per frequency point.
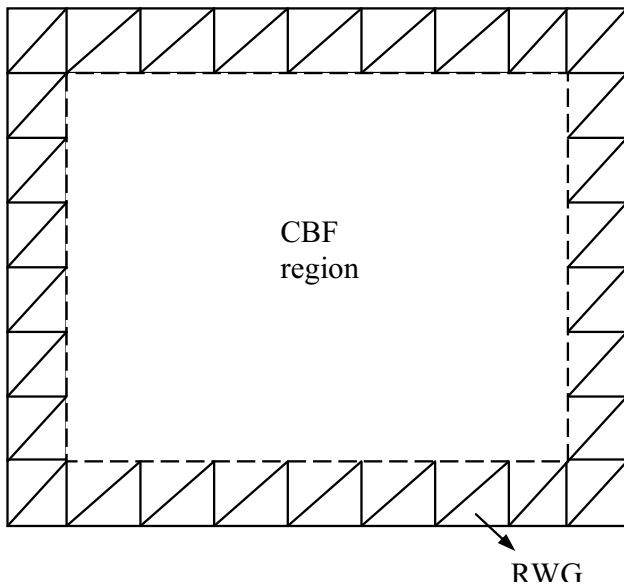

Figure 5a : Hybridization of the basis for a conducting plate by combining the CBF and the RWG basis functions.

Next, we demonstrate a hybrid approach, which combines the CBF and the RWG basis functions for the case

of a PEC plate, as shown in Fig.5a. For the case of 4λ x 4λ PEC plate, the CBF region is 3.8λ x 3.8λ in size, while the RWG bases have been used in the edge regions. The radar cross-section of the plate has been evaluated by using the hybrid approach and compared with that obtained by using the conventional MoM over the entire plate. The results presented in Fig.5b, again show good agreement with each other. They serve to demonstrate that, if desired, the CBF approach can be hybridized with the RWG method in a convenient manner.

As mentioned earlier, we have the option to derive the CBFs by using a legacy MoM code that employs the RWG basis functions. To illustrate this procedure, we used an RWG-based MoM code (which solves EFIE) and derived the CBF's for two specific examples, namely, a 2λ x 2λ PEC plate, and a 90° corner reflector. For the case of the PEC plate, the geometry was divided into 16 blocks, leading to 256 CBFs. It was found later, via numerical experimentation, that only 8 CBFs would have been sufficient for this problem, as compared with 4720 unknowns that were needed in the original RWG discretization. The RCS results for the plate have been computed for the normal and 45° incidence cases, and are presented in Fig.6. The close agreement between the CBF-based and the direct MoM solutions once again serve to demonstrate that the proposed technique yields accurate solutions in a numerically efficient manner.

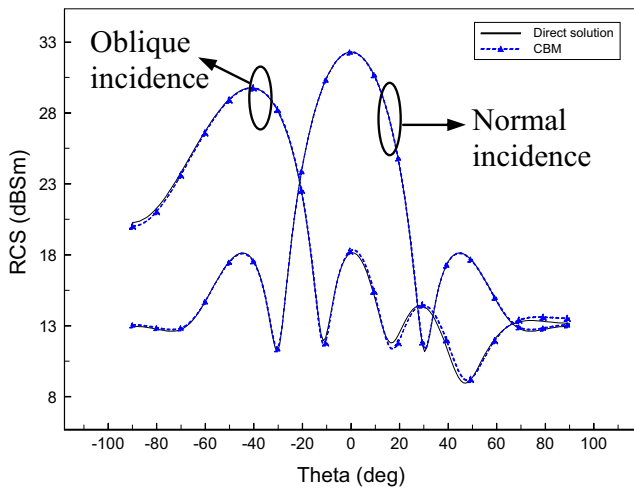For the case of 90° corner reflector, shown in Fig.7, the

Figure 6 : Radar cross-section of 2λ x 2λ PEC plate at normal and oblique incidences.
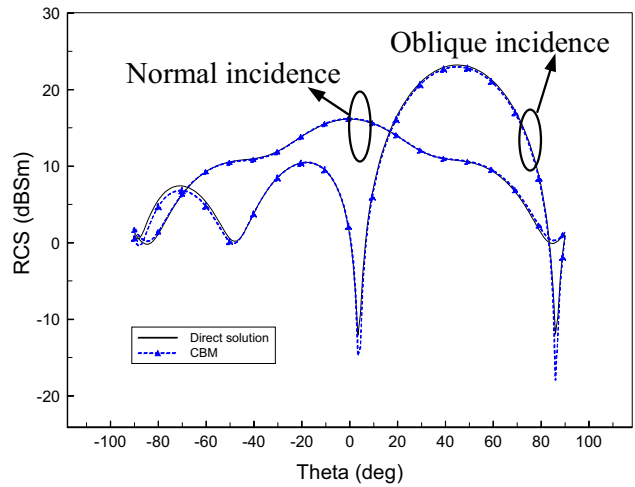

Figure 8 : Radar cross-section of corner reflector at normal and oblique incidences.
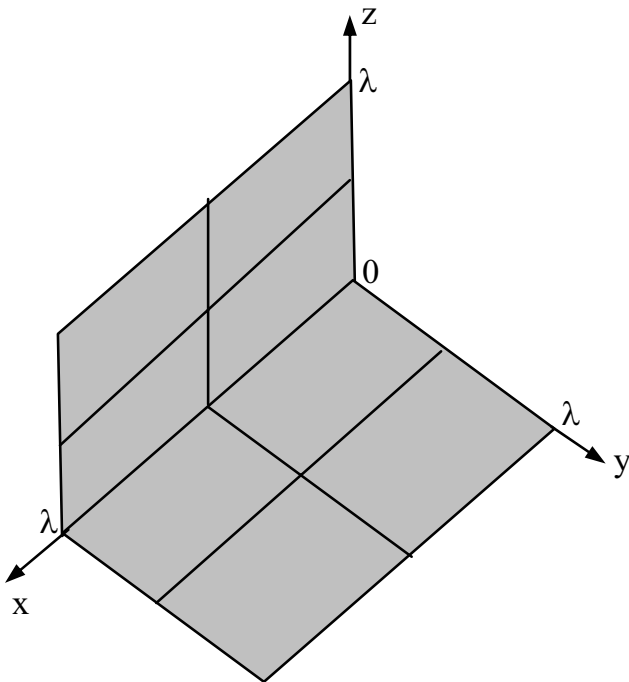

Figure 7 : A 90° corner reflector divided into 8 blocks. Each block has 8 CBFs.

original RWG discretization require 2340 unknowns, but we employ only 64 CBF's with 8 blocks, though the latter could be further reduced to only 3 blocks, *viz.,* the two plates and the overlapping corner region. (The optimal number of domains can be determined by balanc-

ing the times taken to generate the CBFs, constructing the reduced matrix, and solving the same.) The RCS of the structure has been computed for normal and 45° incidence cases, by using the conventional RWGs and the CBFs, and the results are presented in Fig.8. We note that, unlike the plate example, there is considerable amount of interaction (multiple scattering) between the adjacent facets in the present case. The results computed by using the CBF are found, once again, to be in close agreement with the direct MoM solution, and this proves that the concept of CBF's can be extended to the cases where the individual facets strongly couple to each other.

In fact, our experience shows that the CBF approach is very general and, as mentioned earlier, it has been applied to microstrip circuits and antennas, including resonant configurations, *e.g.*, filters and patch arrays, that are highly resonant structures. As for the RCS problems, it has been also used to analyze a number of other objects, including a PEC cube and a large, truncated Frequency Selective Surface (FSS) to test its versatility. Detailed discussions of these problems are beyond the scope of this paper, and they will appear in separate publications that are currently being prepared by the authors. However, we include the numerical result for the RCS of a truncated FSS screen with 25x25 crossbar elements (arbitrarily large number of patches can be treated with the CBF approach without any difficulty), shown in Fig.9, and the time comparisons with an FMM code that are
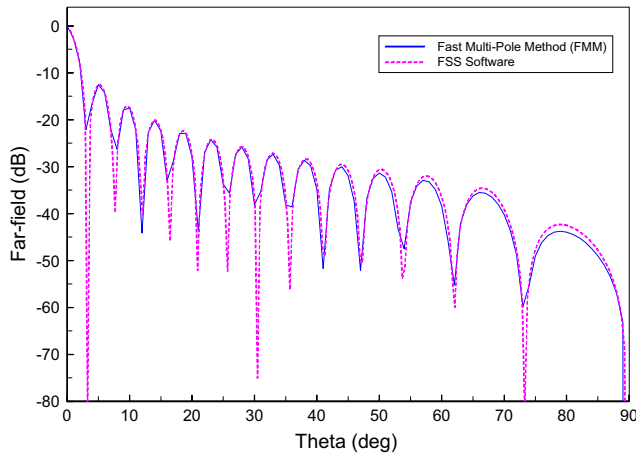
Figure 9 : Scattered Far-field from a 25 x 25 free standing FSS screen.

Table 1 : Computational details

MonoStatic Radar Cross-Section		
Fast Multipole Method	: 19.872 dBSm	
FSS Software	: 19.729 dBSm	
Computational Requirements		
	Memory(Mb)	CPU time (s)	Computer
FMM	1087	25212	IBM RS6000
FSS Code	19	689	PC 662MHz

given in Table 1.

We should point out the convergence of the FMM code was much slower for the FSS case than that for a thin plate of the same size, because the associated MoM matrix for the FSS, which is a resonant structure, was ill-conditioned, even when a pre-conditioner was employed to improve the matrix conditioning, however, no such problem was experienced in the CBF approach. It may be worthwhile to mention that the infinite extension of the truncated FSS is a doubly-periodic of the structure, which can be analyzed conveniently by modeling just the unit cell, which is a very manageable problem.

Finally, we close this section with several comments on the CBFM. First, we mention that CBFM has been recently extended to handle multiple incident angles, where a single set of primary basis functions, corresponding to these angles, have been found adequate without the need of secondary functions. This separates the CBFM

from iterative approaches, which must start the solution process anew for multiple right hand sides (incident angle) [Mittra and Prakash (2003)]. In fact, the CBFM has been used over a wide frequency range as well, with a single set of basis function over the range [Prakash (2003)]. This feature also sets the CBFM apart from other higher-order basis functions. Second, we mention that the CBF approach can be extended to objects covered by one or more layers of coating materials (without the use of impedence boundary condition approximation), curved facets with large radii of curvature that have no shadowing (as mentioned earlier, the shadowing case can be easily treated by using multiple patches), and facets with rough surfaces. Once again, the authors plan to cover these cases in future publications that are currently in preparation.

6   Conclusions

In this paper, we have introduced a novel approach for efficient solution of the problem of radar scattering from object that can be represented in terms of facets. The approach is based on the use of Characteristic Basis Functions (CBFs), which are not bound by the conventional $\lambda/20$ discretization in the context of the Method of Moments, but are defined on much larger-size domains. Representing the induced currents on a scatterer in terms of the CBFs enables one to handle electrically large geometries with a relatively few unknowns, typically several orders of magnitude smaller than that required in the RWG approach. Furthermore, the CBFs can be constructed by using a matrix-free approach that can result in considerable savings in CPU time and memory, even when compared to the FMM approach for solving similar problems. The authors believe that the CBF approach will open new horizons in numerical modeling of electromagnetic scattering problems and would enable us to solve much larger-size problems than currently possible by using one of the existing numerically rigorous codes– be it MoM (including FMM and its multi-level variants), FEM, or the FDTD. And yet, on the basis of our experience, we anticipate that the accuracy levels achieved by the CBF approach is expected to be comparable to those provided by the above codes, and superior to the asymptotic techniques that are typically employed for such problems when the CPU time and memory limitations preclude the use of the rigorous codes.

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