 CMES, vol.5, no.2, pp.171-187, 2004

PMMC cluster analysis

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Abstract: Particle distribution influences the particulate reinforced metal matrix composites (PMMC). The knowledge of particle distribution is essential for material design. The study of particle distribution relies on analysis of material images. In this paper three methods are used on an image of an Al/SiC composite. The first method consists in applying successive dilations to the image. At each step the number of objects and the total object area are determined. The decrease of the number of objects as a function of the area is an indicator of characteristic distances. The second method is based on dilations of one particle among all the others. Then each time it touches a neighbor the number of the step i of the process is recorded and gives the distance to the nth neighbor. This is done for each particle of each image. Thus statistical parameters of the distribution of the distance to the six first neighbors are obtained and compared to the previous characteristics. The third method is the covariance method. These three methods are tested on synthetic images of known characteristics. Then the Al/SiC image is analyzed and once the characteristics are identified a statistically identical image could be created later.

keyword: image analysis, distance, composite, simulation.

1 Introduction

Particulate metal matrix composites improve the mechanical characteristics of the Al material, but their mechanical characteristics are dependant on the particle spatial distribution and specifically on the fact that particles are not distributed at random but tend to be clustered at some scale. Material design could be processed in three steps :

- Simulation of the structure though image analysis and image simulation ;
- Mechanical simulation of tests in order to know the mechanical properties (see for example Ghosh et al (1997) or Tvergaard and Orts Pedersen (2000));
- Material processing in order to obtain the same particle distribution as simulated.

Thus the material can be developed and improved. The first step of this process need a mean to characterize particle distribution. Thus the challenge is to obtain representative characteristics of the particle spatial distribution for future simulations.

Many studies have been made on distance characterization on images. A first group of methods consists in the analysis of the centroid distribution. The particle centroids are studied as a set of points through:

- Radial distribution function H(r), or g(r) which characterize the number of points in circle of r radius centered on each point of the set (Schwartz et Exner (1983), Karnezis et al (1998), Li et al (1999)).
- Dirichlet tessellation which builds, around each centroid, polygons whose borders are all laying at half distance from two neighboring centroids (Spitzig et al (1985), Hermann et al (1989), Fraser (1991), Murphy at al (1996), Bertram and Wendrock (1996), Karnezis et al (1998), Ghosh et al (1997)).

A second group studies the particle distribution directly:

- Finite body tessellation builds, around each particle, polygons whose borders are laying at half distance from two neighboring particles. This allowed border to border distance to be computed (Borgefors (1986), Lafferty (1993), Boselli et al (1999) and Redon et al (1999), Dubois et al (1999)).
- Quadrat method divides the image into contiguous quadrats and makes an analysis of the distribution of local area fractions (Curtis and Mac Intosh (1950), Greig-Smith (1952), Karnezis et al (1998)).

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- Covariance function gives some of the characteristics of a distribution (Serra (1982), Soille (1999), Susagna et al (2000)).

The two tessellation methods allow the computation of the distance to the first neighbor, the mean distance to the other neighbors (around the polygon), the number of neighbors, the polygon area, and the local area fraction in the polygon. The distributions of these characteristics are compared to random set of points/particles (depending on the tessellation type) in order to have some statistical characteristics of the particle distribution. But these methods do not give precise distance values with their signification.

A third possibility has been presented by Yotte et al (2001). It allows the determination of various distances such as intra–cluster inter–particle distance, inter–cluster distance, or isolated particles to cluster distance.

In the following we will compare the results obtained through an improvement of this last method and results obtained by a new method giving the inter–particle distance to the first neighbor and to the five further neighbors. This will be applied to five synthetic images of known characteristics. Then the same study will be made on an Al/SiC composite image. The work is done with Micromorph® image analysis software (Armines/ENSMP, Centre de Morphologie Mathématique, France). Micromorph is a software that computes all the image analysis operations and transformations which are defined in mathematical morphology (Serra, (1982)). The development of routines combining these transformations or operations is possible. The programs described in this work have been written in the Micromorph® programming language.

2 The material

The composite is a SiC particle reinforced commercial aluminum obtained by a powder metallurgy process (Li et al. (1999)). The matrix is an X2080 aluminum alloy (with 3.8% Cu, 1.8% Mg and 0.2% Zr weight percentages) with 15 % volume fraction SiC particles. This material is used in automotive and aerospace applications and presents high thermo–mechanical properties.

The image studied here is x100 magnification. Its characteristics are shown on Tab. 1. Image is 640x512 pixels, each pixel corresponding to a gray level in the range [0-255], and to a size 0.63 μm. The image is presented on Fig. 1.

Table 1 : Characteristics of the image.

Name	Magnification	Number of particles	Particle area	Mean particle area (μm^2)
Image 5	x 100	470	62926	54

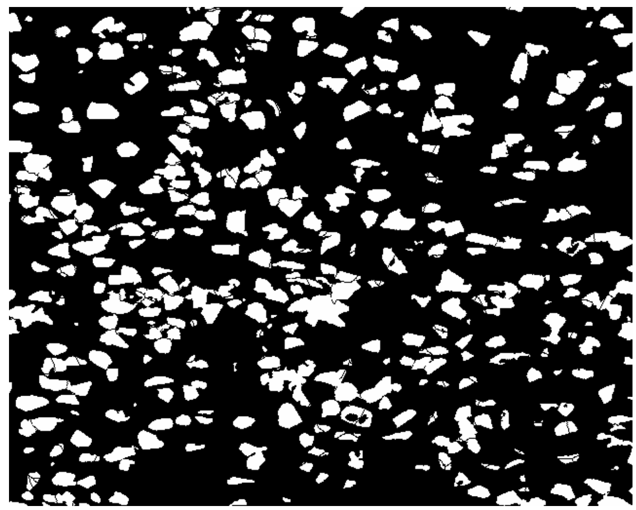


Figure 1 : Binary image of the Al/SiC composite.

The final objective of the paper is to characterize this material, through this image. In the following image analysis will be used in order to

- create synthetic images of known characteristics,
- characterize inter–particle distance on these images with three different methods,
- compare the various results,
- apply this study to the Al/SiC image.

3 How to characterize inter–particle distance?

In this part we will present three methods used for the evaluation of the inter–particle distance. Some important morphological operators must first be explained. So we will explain what are a dilation and an erosion (Serra (1982), Soille (1999)) and then present the method used to create images.

3.1 Image analysis basic morphological operators

3.1.1 Dilation

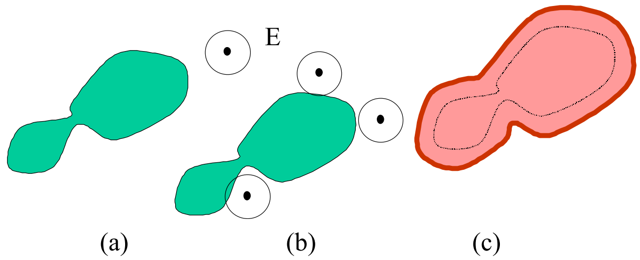


Figure 2 : Dilation of an object by a disk. The objet A, in grey, is shown in (a) with the disk beside. The object is tested by the disk in (b), and the resulting dilated object is shown in (c).

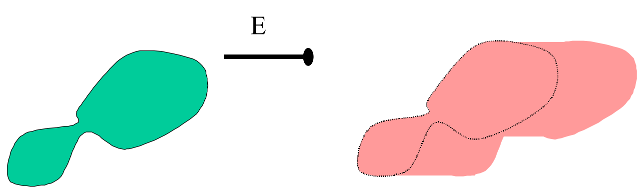


Figure 3 : Dilation of an object by a line.

Let be an object A and a structuring element E. A structuring element is a small geometrical figure used as a tool for the transformation; it is characterized by its shape, its size and its center. The element E can be set at any point of the image. Then E is tested on the object A. If the element E touches to A then the center (origin) of E belongs to the dilated of A. On Fig. 2, E is a disk. We will use another structuring element which is a line whose center (or origin) is defined on its right extremity. Fig. 3 presents the directional dilation.

A dilation is characterized by its size (number of steps elementary dilation is repeated) and by the structuring element.

3.1.2 Erosion

It is the dual operation (Fig. 4). Let be an objet A and the structuring element E. Then E is tested on the object A. If E is completely included in the object A then the center (or origin) of element E belongs to the eroded of A.

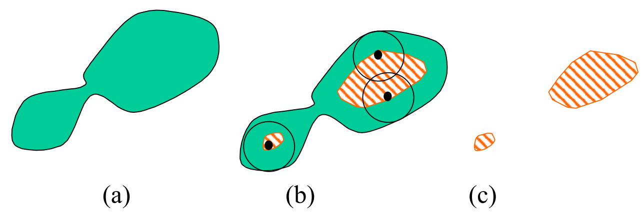


Figure 4 : Erosion of an object by a disk. The objet A, in grey, is shown in (a). The object is tested by the disk in (b), and the resulting eroded set of two objects is shown in (c).

3.1.3 Conclusion

The aim of this paper being to characterize clusters of particles it would be interesting to use euclidian distance, see Borgefors (1986), but we are working with discrete images and such a distance does not work well. We just need an accurate tool to characterize and compare the spatial distribution of particles laying on various images. So we decide to use distance function defined on a hexagonal grid; in such a way two particles are dilated by a hexagon (structuring element) of increasing size until they became connected and the distance is assumed to be the size of the structuring element. Fig. 5 shows the hexagonal grid defining the pixel relative position (nodes of the grid), and presents also the three directions of the grid we used here for the directional dilation. The hexagon is the structuring element for which all the pixels lay at the same distance from the center, as the disk in the euclidian plane. The result of the dilation by such an element is shown on the figure too.

3.2 Definitions

3.2.1 Function F

Two characteristics can be estimated on the image: the number of objects and the object area. From those two measures we want to obtain the image cluster characteristics.

Successive directional dilations (Fig. 3) in one precise direction of the hexagonal grid (direction 1, 2 or 3 see Fig. 5), with a size 1 dilation, modify the number of objects in the image. All the particles are dilated together. As each particle grows, it reaches the nearest particle when its dilation size is equal to the distance to this neighbor. Then there is one single object instead of two. Thus, at each

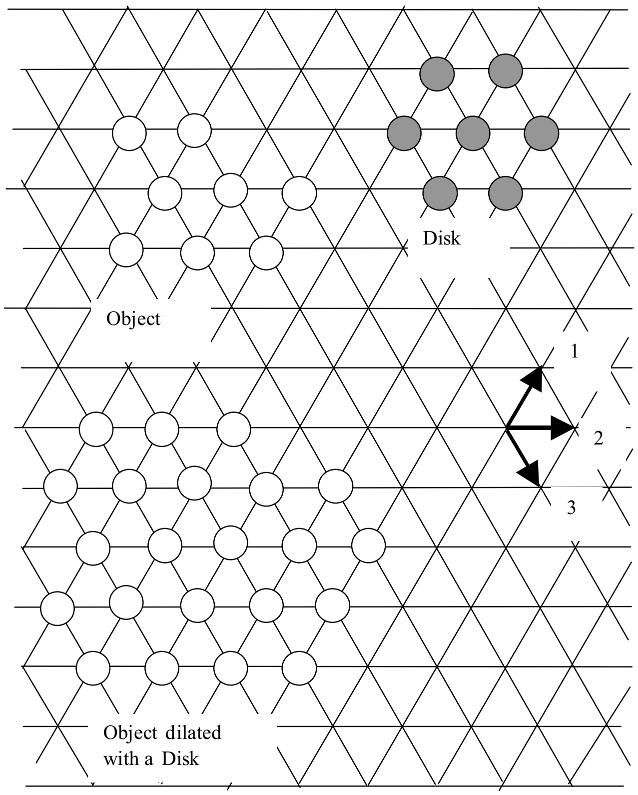


Figure 5 : Discrete image on A hexagonal grid: the result of a dilation by a disk is shown as well as the three first directions of the hexagonal grid. The pixels are represented as circles and the lines represent the grid.

dilation, the number of objects in the image decreases and the rate of decrease is related to the distribution of distances between particles. This could be indicative of other characteristic distances of the image such as inter–cluster distance. This operation is done for the three directions of a hexagonal grid.

At each step i, the number of objects and the total object area are computed. This is done until the object size is no more growing. Equation 6 gives the curve indicative of the number of objects decrease as a function of the occupied area.

$$\frac{\frac{N_i - N_{i+1}}{N_0}}{\frac{A_{i+1} - A_i}{A_f - A_0}} = F(d_i) \tag{6}$$

With:

N_i: number of objects at the step i;

N_0: number of objects initially;

A_i: area occupied by all the objects at the step i;

A_f: area occupied by all the objects at the end of the operation;

A_0: area occupied by all the objects at the beginning;

D_i: dilation size.

The initial number of objects is the number of objects before any dilation. The initial area is the area occupied by the particles before any dilation, the final area is the area occupied by the dilated particles when no more dilation is possible.

3.2.2 First neighbor distance

The particles are isolated one after the other. Then successive dilations of size 1 are applied to the image containing the particle. This dilated particle is added to the initial image and the number of objects is determined. If this number decreases it means that the inter–particle distance is equal to the number of dilations: the dilated particle touches other particle and forms with them one unique object. At each decrease of the number of objects, is associated a distance to the further neighbor. In order to avoid border influence only particles which are at a 30 pixels distance from the image border are studied.

3.2.3 Covariance

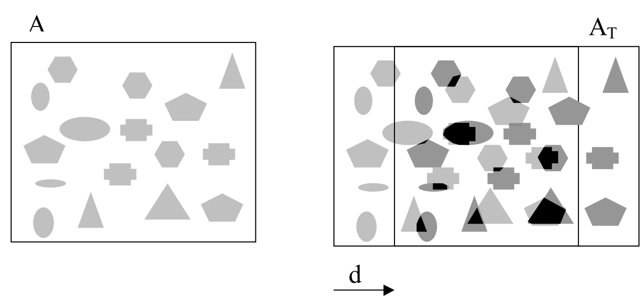


Figure 6 : Image A and the comparison between the translated of A, A_T and A: the objects of A are in light grey and A_T objects are in dark grey. The pixels belonging to both A objects and A_T objects are in dark.

Let be a binary image A. This image is translated into an image A_T, at a distance d from A and in a given direction. The two images are compared. The pixels belonging to objects on A and on A_T are counted and this area gives the covariance of A for the distance d (Fig. 6).

3.3 Clustered random images

The objective is to create images where clusters exists, and then to evaluate the various methods on these images. Measurements must give the same characteristics as those that have been imposed during the image creation process.

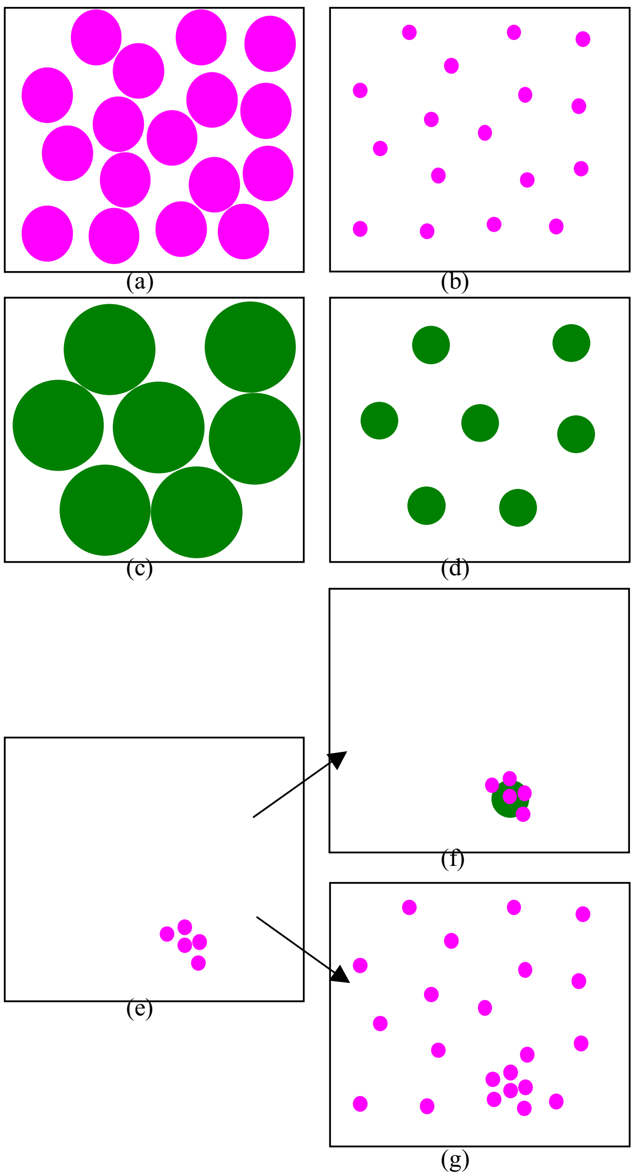


Figure 7 : Creation of a clustered image; (a), (b) first phase of particle laying out of clusters; (c), (d) clusters, (e), (f) and (g) second phase of clustered particles.

3.3.1 Creation method

The process follows two steps. A first step creates particles which do not belong to clusters (first phase particles) and a second one creates particles belonging to clusters (second phase particles), see Fig. 7 (a) to (g).

For creating the first phase particles, random disks of d_1 diameter are created. These disks do not touch each others (Fig. 7 (a)). Diameter d_1 is chosen so that the image can contain all the disks. They are at least at one pixel distance from each other. Then they are eroded to d_p diameter which is the final particle size (Fig. 7 (b)). Thus the minimal inter–particle distance is:

$$d_2 = 2.\left(\frac{d_1}{2} - \frac{d_p}{2}\right) + 1 = d_1 - d_p + 1, \tag{2}$$

which is shown on Fig. 7.

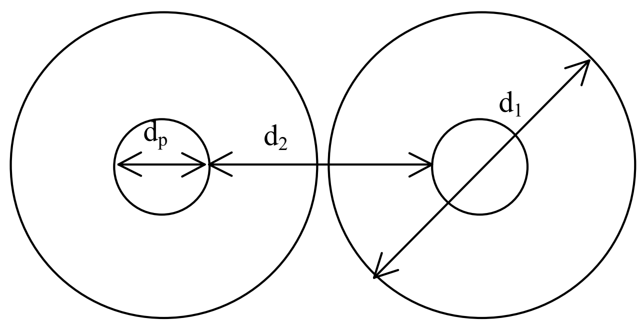


Figure 8 : Distance between unclustered particles

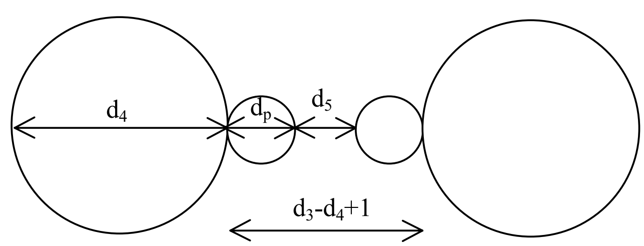


Figure 9 : Distance between clustered particles

For creating the second phase particles, random disks of d_3 diameter are created independently of the first phase particles (Fig. 7 (c)). These disks do not touch each others. Then they are eroded to a d_4 diameter (Fig. 7

(d)). This ensures a minimal distance of d_3-d_4+1 between their borders. Then particles (diameter d_p) are created (Fig. 7 (e)) which

- have to touch or be inside the previous disks (Fig. 7 (f))
- and must not touch the first phase particles (Fig. 7 (g)).

Then the minimal inter–cluster distance is:

$$d_5 = 2.\left(\frac{d_3}{2} - \frac{d_4}{2}\right) + 1 - 2d_p = d_3 - d_4 + 1 - 2d_p, \quad (3)$$

as shown on Fig. 9.

3.3.2 Image characteristics

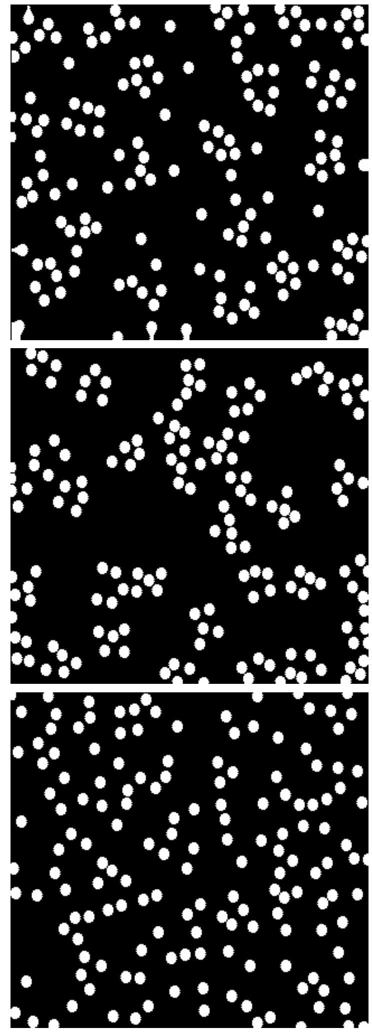
Clustered images are created. The particles have d_p=11 pixels diameter. Tab. 2 gives the images characteristics.

n_1 is the number of particles of the unclustered phase, n_2 is the number of clusters and n_c is the number of particles in each cluster.

Table 2 : Image characteristics

Image	n_1	n_2	n_c	d_1	d_2	d_3	d_4	d_5
Cluster 1	51	22	5	33	23	57	33	3
Cluster 2	51	22	5	33	23	57	33	3
Cluster 3	51	22	5	33	23	57	33	3
Cluster 4	31	26	5	49	39	49	33	1
Cluster 5	1	32				41	25	1

The first three images are statistically identical since they have been created with the same parameters. The two others are different: “Cluster 4” has more clusters and less isolated particles. The clusters are nearer from each others, the minimum distance is thus 1 pixel (as d_5 is found negative), and the maximal distance (d_3-d_4+1) is 17 pixels. “Cluster 5” is made of 32 clusters, just one particle laying outside. All the images have the same number of particles (161). Fig. 10 ((a) to (c)) shows image “Cluster 5” and “Cluster 1” as well as a random image with 161 identical particles.



(a) “Cluster 1”

(b) “Cluster 5”

(c) Random image

Figure 10 : created 161 particle images: (a) 22 clusters (b) 32 clusters (c) random image

3.4 General method

3.4.1 Comparison

For each method, the result for the three containing cluster images must be compared to the theoretical data. The distance for which there is a significant difference of result between the theoretical random distribution and clustered particle distribution is a characteristic distance.

There are two problems:

1. The comparison is done between the results of clustered image and the theoretical results of random images. How can we say that the difference is an accident or if it is due to the clusters? We choose a probability p of error and then the probability for the clustered result not to differ from the random result is :

$$\text{Prob}(\mu - u_p\sigma < X < \mu + u_p\sigma) = 1 - 2p \quad (4)$$

where u_p is given by the normal law tables (we assume that the random image distance distribution follow a normal law, of mean μ and standard deviation σ) and X the interparticle clustered distance. The interval [μ-$u_p\sigma$, μ+$u_p\sigma$] is the p% probability interval. One such interval is defined for each distance. Then the characteristic distances of the distribution are those for which the clustered image results are not in the confidence interval.

2. There is no theoretical data for inter particle distance distribution in the case of particles of any shape. There is no analytical solution. Thus the law giving inter–particle distance in hard core images has to be estimated through results from N random image simulations. What is the better value for N? "Better" means here the values of μ and σ respectively mean and standard deviation of the normal law, related to the random image distance, have to be estimated. Thus the confidence interval is an estimation of the real one with a chosen risk of error α%.

3.4.2 How many images are necessary?

Let be X_{th} the parameter for which we want an estimation. X_{th} could be the distance function we call F (Yotte et al (2001)), the number of neighbors or the covariance result for a given distance d. Since we need to estimate an optimal number N of images we assume it can be inferred from

$$\text{Prob}(|m \pm ks| < |\mu \pm u_p\sigma|) = \alpha \qquad (5)$$

where:

- μ and σ are the mean and the standard deviation of a normal variate X,
- m and s are the estimate for μ and σ from an N sample of this distribution,
- and u_p is related to the interval of probability:

$$\text{Prob}(\mu - u_p\sigma < X < \mu + u_p\sigma) = 1 - 2p$$

The parameter k depending on p, α and N is tabulated; since
$k = \dfrac{t}{\sqrt{N}}$ the optimal size of the sample can be estimated.
For p being 10%, α being 25 % and N being 20, statistic tables give a coefficient k (N, p, α) of 1.93. Thus we will take this value for the following study and use 20 random images to estimate the mean and standard deviation. For this data set, if on a given image and a given distance d,

$$X < m_{th} - 1.93\sigma_{th} \text{ or } X > m_{th} + 1.93\sigma_{th} \qquad (6)$$

we will conclude that d is a characteristic distance for this image. The mean m_{th} will be estimated from 20 images.

3.5 Function F results

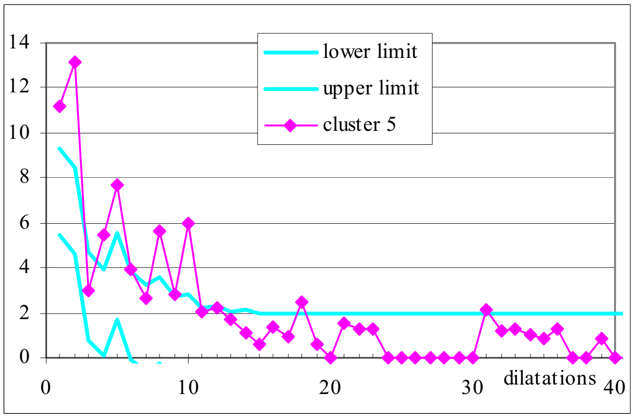
Fig. 11 shows results for "Cluster 5" direction 1 (Fig. 11 a), direction 2 (Fig. 11 b) and direction 3 (Fig. 11 c). Three curves are plotted for each image: the mean divided by the standard deviation minus 1.93 and the mean divided by the standard deviation plus 1.93, and the result of the image divided by the standard deviation. We choose to plot $\frac{m}{\sigma} \pm 1.93$ instead of $m \pm 1.93\sigma$ because the standard deviation value σ vary with the dilation size. This choice ensures a constant difference of 3.86 between the upper limit and the lower limit.

Some points are outside the limits and are said to be representative of structures on the image. We decide that only series of consecutive points or isolated points which appear in at least two directions are representative points.

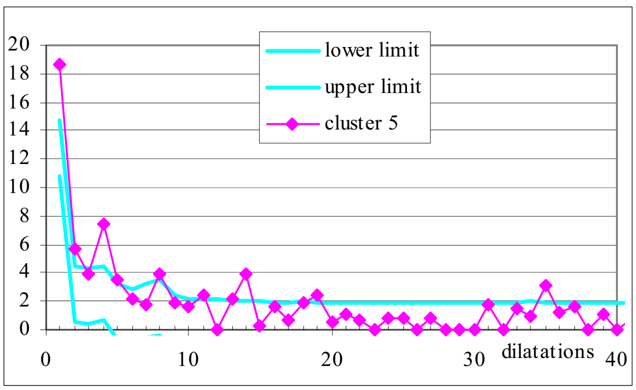
Thus Fig. 11a shows that 1 and 2 pixels are characteristic distances as well as 4 and 5 pixels for example. If a minimal distance is searched then the first number of the series is the distance, if it is a maximal distance then the last number of the series is the distance. On Fig. 11a the first series is [1-2]. Thus 1 pixel is the minimal intra–cluster inter–particle distance. The second series is [5-6], thus 6 is the maximal intra–cluster inter–particle distance. The three directions give similar results [1-2] then [5-6] for direction 1 and 3 and [4-5] for direction 2. All the other distances laying out of the limits are not taken as characteristics.

Tab. 3 shows the results related to all the images. The last column gives the common results (series and single values if they appear in two directions at least).

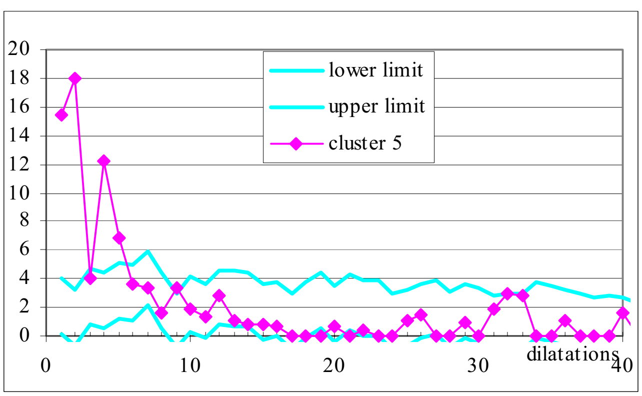
- "Cluster 1" image seems to have 1 as characteristic distance (first of the series [1 to 5]). As this series is rather long we suppose that 5 the last of the series could be a maximal characteristic distance. Also 7, 11 and 21 (firsts of the next series), as well 19 pixels seems to be characteristic distances.
- In the case of "Cluster 2" image which have been created with similar parameters, 1, 10 and 32 pixels



(a) direction 1



(b) direction 2



(c) direction 3

Figure 11 : "Cluster 5" results, a in direction 1, b in direction 2 c in direction 3.

seems characteristic with no value between 1 and 10.

- "Cluster 3" image is similar to "Cluster 1" with 1, 4

Table 3 : Results related to the 5 images "Cluster"

	Direction 1	**Direction 2**	**Direction 3**	**Characteristic distances**
"Cluster 1"	[2-3] [7-8-9] 11 [22-23-24]	[3-4] [11-12] 19 [21-22]	[1-2-3-4-5] 7 19	[1-2-3-4-5] [7-8-9] [11-12] 19 [21-22-23-24]
"Cluster 2"	[1-2] [10-11-12-13] 33	[1-2] [12-13-14] [32-33]	[1-2] [13-14]	[1-2] [10-11-12-13-14] [32-33]
"Cluster 3"	2 4 [7-8-9] 12 14 [21-22] 32	[4-5] 12 14 [31-32]	[1-2] 4 7 14	[1-2] [4-5] [7-8-9] 12 14 [21-22] [31-32]
"Cluster 4"	[4-5] 7 10 [14-15] [20-21-22-23]	1 5 [13-14] [16-17-18] [31-32]	[1-2-3-4] 7 [16-17-18] 31	[1-2-3-4-5] 7 10 [14-15] [16-17-18] [20-21-22-23] [31-32]
"Cluster 5"	[1-2] [4-5-6] [8-9-10]	[1-2] [4-5] 8 19	[1-2] [4-5] 9 19	[1-2] [4-5-6] [8-9-10] 19

(or 5 as in "Cluster 1"), 7, 12, 14 21 and 31 pixels as characteristic distances.

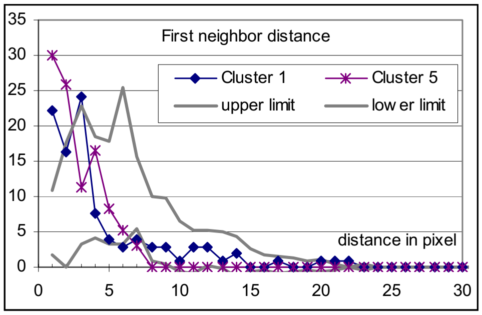
- "Cluster 4" image which has more clusters and less isolated particles than the three previous images, has 1 and perhaps 5 (as in "Cluster 1"), 7, 10, 14 16 or 18, 20 or 23 and 31 pixels as characteristic distance.

These results will be compared to those obtained with the neighbor method on the same images.

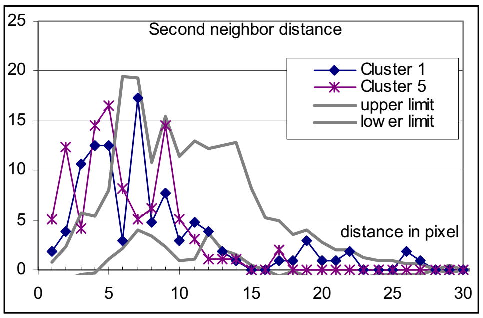
3.6 First neighbors results

Fig. 12 a to f gives the frequency in percentage of the distance to the six first neighbors for images "Cluster 1" and "Cluster 5". An image curve is either over the upper limit, either inside the two limits either under the lower limit. When it is over the upper limit, this means that some particles have more neighbors at this distance than random images. Thus the first distance of the series is a minimal characteristic distance. When the curve is under the lower limit, this means that some particles have less neighbors at this distance than random images. Thus the first distance minus one of the series is a maximal characteristic distance.

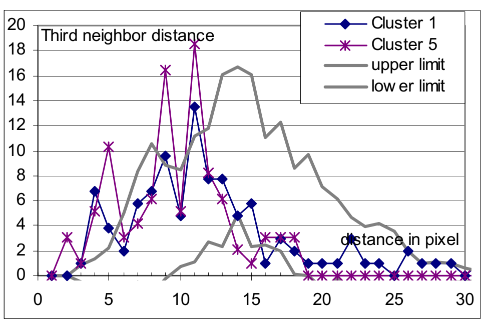
For example in the case of image "Cluster 1" on fig.12 (a), first neighbor distance, 1 and 3 pixels are distances at which a particle has more chances to have a neighbor than in a random image. More particles are nearer from other particles in "Cluster 1" than in random images, thus



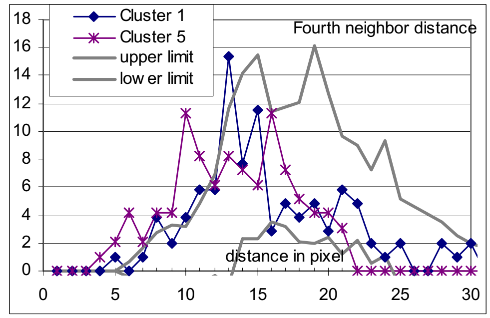
(a) First neighbor distance



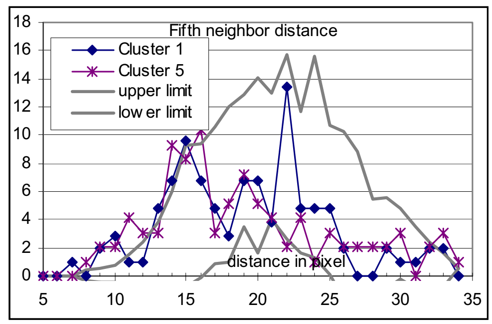
(b) Second neighbor distance



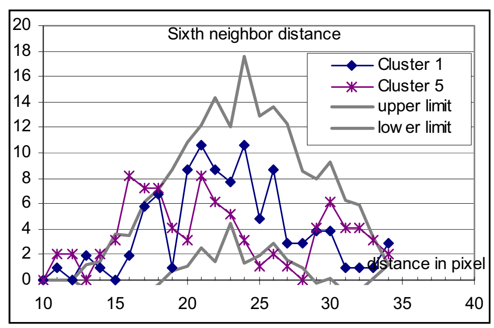
(c) Third neighbor distance



(d) Fourth neighbor distance



(e) Fifth neighbor distance



(f) Sixth neighbor distance

Figure 12 : Frequency of the neighbor distances for images “Cluster 1” and “Cluster 5”.

“Cluster 1” seems to be more clustered than a random image. The second characteristic distance is 7 pixels: it is under the lower limit. Up to 6 pixels the particles have more or as many neighbors as in the random image. But at 7 pixel distance lay less first neighbors than in a random image. Thus particles which have first neighbors at 1 to 6 pixels are in a cluster and those of which first neighbors are at more than 7 pixels lay out of clusters.

The set of the first six neighbors curves (fig 12 (b) to (f)) gives perhaps more information but it is difficult to find which distance is really characteristic. Our clustered images have 5 particle clusters. Thus the fifth and sixth neighbor frequencies must lay inside or under the two limits. It is rather true for the two images “Cluster 1” and “Cluster 5”, just 1 or two particles (1 to 2% frequency) are laying very near from a cluster. The second, third and fourth neighbor plots shows higher differences between upper limit and image curves for high distances which is logical. Thus the lower difference between the upper limit and the curves is an indication of the number of clusters of particles.

“Cluster 5” particles are all laying in clusters. Only 68% of “Cluster 1” particles are planned to lay in clusters. Thus for the four first neighbors, the characteristic dis-

Table 4 : "Cluster" image characteristic distances

	Neighbor 1	**Neighbor 2**	**Neighbor 3**	**Neighbor 4**	**Neighbor 5**	**Neighbor 6**
"Cluster 1"	1$^+$ 3$^+$ 7$^-$ [21-22]+	[1-2-3-4-5]$^+$ [13-14]$^-$ [26-27]$^-$	[3-4-5]$^+$ 9$^+$ 11$^+$ 14$^-$ 16$^-$	5$^+$ 8$^+$ [10-11] 13$^+$ 16$^-$ [32-33]$^+$	7$^+$ [9-10]$^+$ [13-14]$^+$ 21$^-$ 33$^+$	[11-12]$^+$
"Cluster 2"	1$^+$ [7-8]$^-$ 16$^+$ 19$^+$ 24$^+$	[1-2-3-4]$^+$ 12$^+$ 24$^+$	[3-4-5]$^+$ 10$^+$ 14$^-$ 27$^+$	[4-5-6]$^+$ 9$^+$ 11$^+$	[6-7-8-9-10-11]$^+$	14$^+$ 21$^-$ 23$^+$
"Cluster 3"	1$^+$ 9$^-$ [23-24]$^+$ 27$^+$	[2-3]$^+$ 8$^+$ 11$^-$ 13$^-$ [23-24]$^+$ 30$^+$	[2-3-4-5]$^+$ 14$^-$ 17$^-$ 25$^+$ 17$^+$ 30$^+$	[6-7-8]$^+$ [11-12]$^+$ [21-22]$^-$	[9-10]$^+$ 12$^+$ 21$^-$ 30$^+$	26$^-$
"Cluster 4"	1$^+$	[1-2-3-4-5]$^+$ 8$^+$ 28$^+$	3$^+$ 5$^+$ 10$^+$ 18$^+$	6$^+$ 22-	9$^+$ 12$^+$ 20$^+$ 21-	14$^+$ 20$^+$ 25$^+$
"Cluster 5"	[1-2]$^+$ [7-8-9]$^-$ 12$^-$	[1-2]$^+$ [4-5]$^+$ [12-13]$^-$	[1-2-3-4-5]$^+$ 9$^+$ 11$^+$ [14-15]$^+$	[4-5-6-7-8-9-10-11]$^+$ 16$^+$ [22-23-24]$^-$	[8-9-10-11-12]$^+$ 14$^+$ 16$^+$ [21-22]$^-$ 33$^+$	[11-12]$^+$ 14$^+$ [16-17]$^+$ [25-26-27-28]$^-$

tances of "Cluster 5" (intra cluster inter particle distance and inter cluster distance) have a higher frequency, more particles are laying at this distance from another particle.

Tab. 4 shows the results for the five images. The values are followed by a plus character if the point is over the upper limit and with a minus character if the point is under the lower limit. Series are in brackets.

The three first images are constructed with similar parameters. The three images have 3 characteristic distance for the first neighbor: 1 pixel (value over the upper limit), between 7 and 9 pixels (value under the lower limit) and between 21 and 24 pixels (value over the upper limit). Only the first distance appears in "Cluster 4" and the two first in "Cluster 5". These two images have more clusters than the three first image which could means that the two first distances are representative of clusterization and the third distance is more related to isolated particles.

3.7 Covariance results

The covariance is calculated for the 5 images and the 20 random images which give the upper and lower limits. Fig. 13 gives the various results.

The covariance curves from the three images lay out of the two limits for distance from 4-5 pixels to 15-16 pixels. More particles are laying at distance varying between 4 and 16 pixels from each other than in a random image. This result is due to clustering. There is no great difference between the results in the three directions, which is quite logical as the images are isotropic. Statistically similar images as "Cluster 1, 2 and 3" give covariance curve that are quite similar. Fig. 14 shows the comparison for direction 1 of "Cluster 1, 4 and 5" covariance. "Cluster 5" curve is over the two others for distance values between 6 and 21 and is under for distance between 26 and 37. This means that more particles are at inter–particle distance between 6 and 21 and less are laying at 26 to 37 pixels distance than in the two other images. Thus it is possible with the covariance to see a difference between "Cluster 5" and the other images but the difference between "Cluster 1, 2, 3 and 4" is hard to highlight.

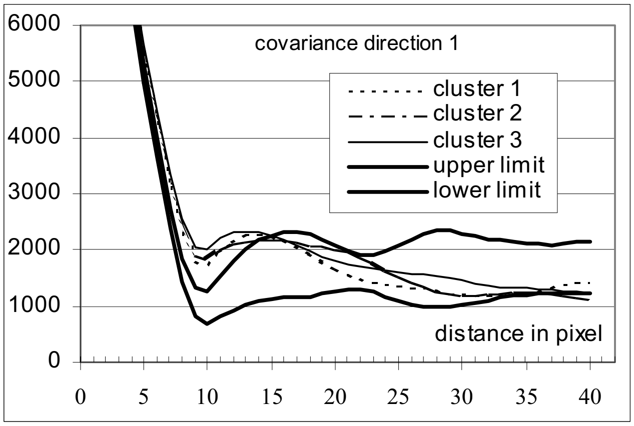
4 Discussion

4.1 Inter–particle distances

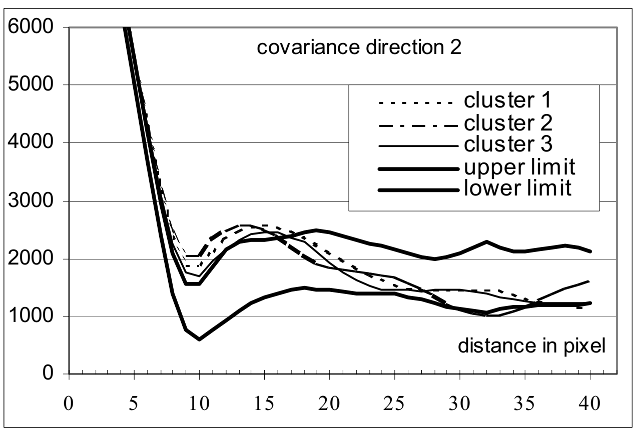
From the neighbor method and the distance function F method we will determine the intra–cluster inter–particle minimal and maximal distances. Tab. 5 compare the results obtained with the two methods applied to the five images. For the first neighbors result when a series were under the lower limit, the characteristic distance is the number just before the first value of the series.

From this table we determine the minimal inter–particle distance d_{min} and the maximal one d_{max} inside a cluster.

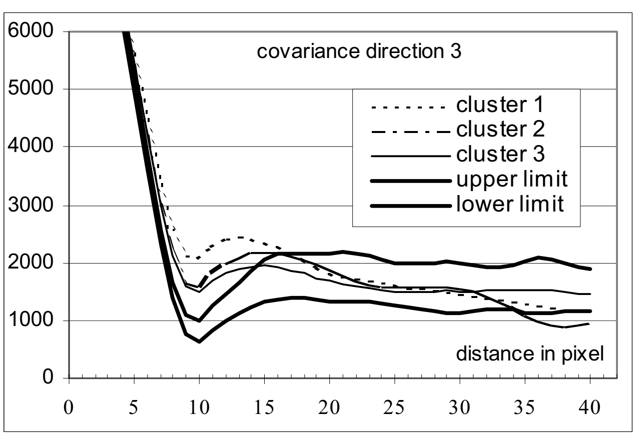
In all the images d_{min} is well identified by the two methods as 1 pixel. The same agreement does not happen for



(a) direction 1



(b) direction 2



(c) direction 3

Figure 13 : covariance curves for clusters 1, 2 and 3 in direction 1 (a), (2) and (3). The three curves are compared to the results of 20 random simulation which give the upper and lower limit.

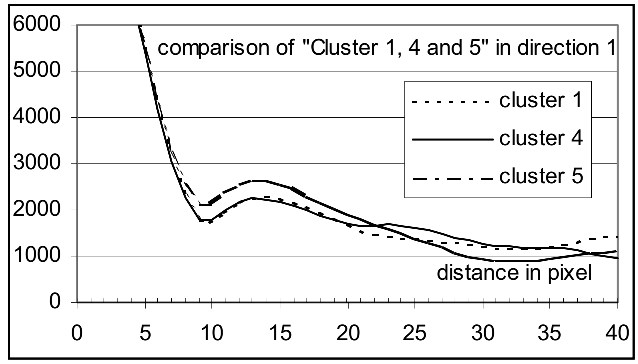


Figure 14 : Comparison of the covariance obtained for “Cluster 1”, “Cluster 4” and “Cluster 5” in the first direction.

Table 5 : Comparison of the results for the two first methods.

	F function	**First neighbor**	**d_{min}**	**d_{max}**
“Cluster 1”	1 5 7 11 19 21	1 (or3) 6 21	1	6
“Cluster 2”	1 10 32	1 6 16 19 24	1	6
“Cluster 3”	1 4 (or 5) 7 12 14 21 31	1 8 23 27	1	5
“Cluster 4”	1 5 7 10 14 16 (or 18) 20 (or 23) 31	1	1	5
“Cluster 5”	1 6 8 (or 10) 19	1 6 11	1	6

the value of d_{max}. It is due to the fact that all the clusters on an image and not at the same distance from each other. In the case of “Cluster 1”, F function gives 5 or 7 for d_{max} and the neighbor method 6. As these value are very similar we take 6 as d_{max}. The distance function F gives for “Cluster 2” a single possibility for d_{max} which seems very high, thus we choose to keep the value given by the neighbor method. Image “Cluster 3” have 5 as d_{max} from the F function and 8 from the first neighbor, thus we choose 5 as d_{max} value. No value is given in the case of “Cluster 4” by the neighbors. F function result is 5. Thus we choose 5 as d_{max}. The image “Cluster 5” shows no such problem as the two methods are in agreement.

Thus when a inter–particle distance have a high frequency it is well identified by the two methods. It is the case of d_{min} for all the images and of d_{max} in the last image where all the particles are clustered and the result is not corrupted by isolated particles.

For the following determination of the inter–cluster dis-

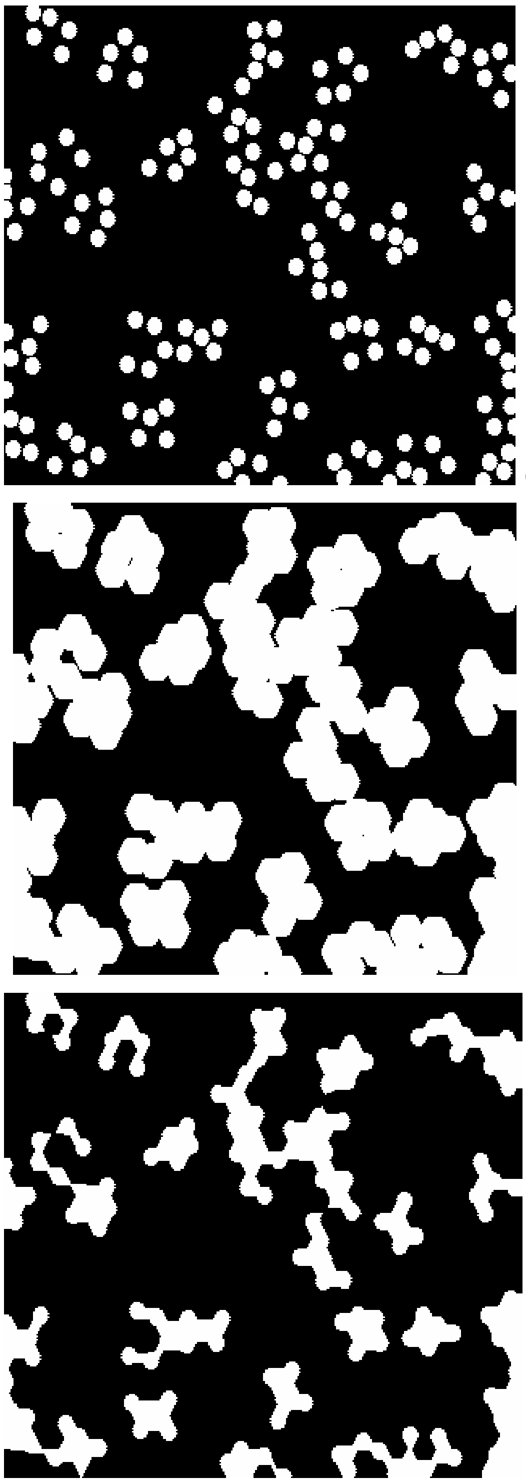


Figure 15 : Cluster construction:
(a) Initial image (cluster 5)
(b) Image after a 5 pixels dilation
(c) Image after 5 pixels erosion and filling of the holes.

tance, the intra–cluster inter–particle distance is taken as 6 pixels for "Cluster 1" and "Cluster 5", and 5 pixels for "Cluster 4".

4.2 Inter–cluster distances

At a first step a dilation with a disk, of the size of the previously identified inter–particle distance, is applied to the image. Thus the inter–particle distance is filled with white pixels and an object "cluster" is build (Fig. 15 b). As all the particles grows together the cluster we build is larger and overlap more particles than the initial one. An erosion is done so that the particles and the clusters recover their initial size again. It is not a perfect because few particles remains bound to the clusters, but their amount is negligible in this study. Close holes appear inside some clusters. Their are filled and the result is shown on Fig. 15 c. The clusters are identified and appear as objects on the image.

The same method that has been used for neighboring particles (§351) is used and the first neighbor cluster distance curves are plotted. Fig. 16 gives the results for the three images "Cluster 1" (Fig. 16a), "Cluster 4" (Fig. 16b), "Cluster 5" (Fig. 16c). The characteristic distance are 12, 14, 17, 22 pixels in the first plot (Fig. 16a). For each distance the number of clusters outside the limits are not the same but they are rather low (2 clusters maximum); and the total number of clusters is low too: 5 clusters for distance 12, 3 clusters for distance 14 and 17, and 2 for distance 22. Thus there is no unique inter–cluster distance, but a distribution. "Cluster 4" shows 4 characteristic distances but only one (6 pixels) is very different from the random distribution. In the last case ("Cluster 5", Fig. 16c) the 21 pixel distance seems characteristic but the cluster distance seems to be fast uniformly distributed.

The construction of the image gives for maximal inter–cluster distance d_3-d_4+1. Tab. 6 gives the comparison of the maximal and minimal distance to the distance found with the neighbor method in the previous paragraph.

We will now compare the distances used for the image creation (see Tab. 2) and these characteristics we found here in order to evaluate the methods for inter particle distance and inter cluster distance determination.

The distances found are all in the given interval except for the last image. This means that

- first the neighbor method is sufficient to identify the

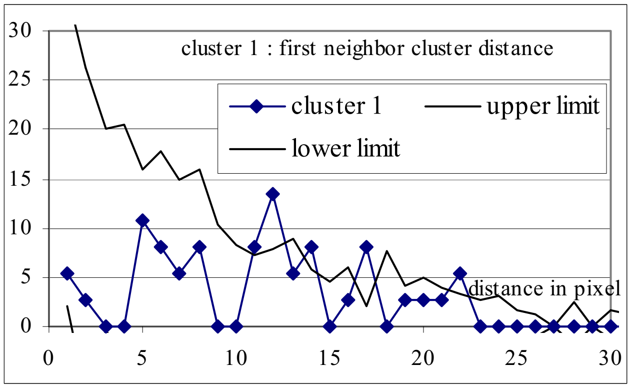


figure 16 (a)

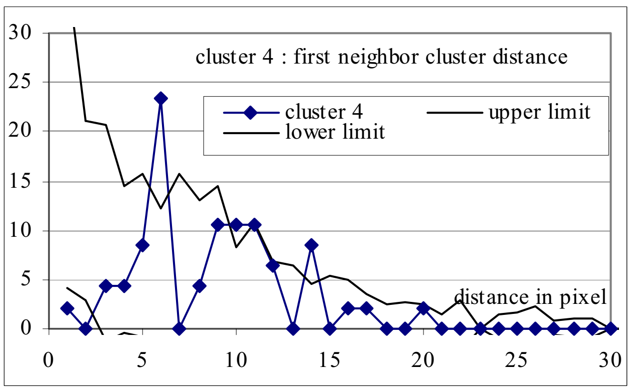


figure 16 (b)

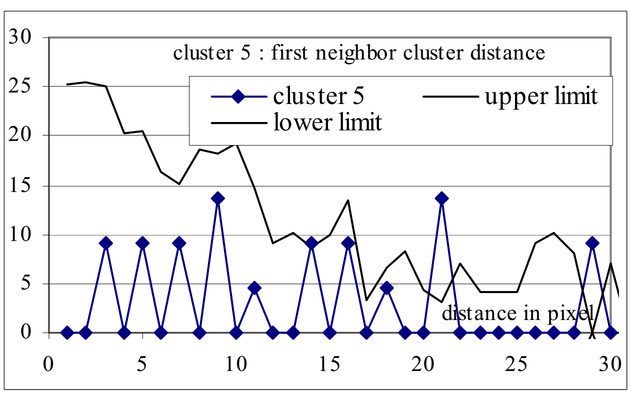


figure 16 (c)

Figure 16 : First neighbor distance to clusters for "Cluster 1" (figure 16a), "Cluster 4" (figure 16b), "Cluster 5" (figure 16c)

Table 6 : Comparison of the construction values of inter–clusters distance with results

	Distance interval for clusters	**First neighbors**
"Cluster 1"	3 – 25	12-14-17-22
"Cluster 4"	1 – 17	6-10-11-14
"Cluster 5"	1 – 17	14-21-29

intercluster distances,

- then the chosen maximal inter–particle distance dmax were relevant.

The plots shows that the distance distributions lays in the creation interval except in the case of "Cluster 4" where the 20 pixel distance could be an isolated particle to another, and for "Cluster 5" where 5 clusters upon 32 are at a greater distance. In this cases the initial d_3 diameter disks used to create the cluster were not of sufficient size to cover the all image.

4.3 Conclusion

Three methods for characterizing inter–particle distance have been tested.

- The intra–cluster inter–particle minimal distance is identified identically in the five images by the F function and the neighbors method. The intra–cluster maximal distance is identified in 4 images for each of these two methods. The inter–cluster distance is not identified here. Then the two first methods give as good results for identifying the two first characteristic distances.

- The inter–cluster distance is too distributed, but the values obtained with the neighbor method are in agreement with those used for the construction of the image.

- The fifth neighbor characteristic distances contain the inter–cluster distances obtained after. But we look at this particular distance distribution because the number of clusters of particles is known which is not the case generally.

- The covariance shows differences between clustered images and random ones, and between the last image and the four others. But it does not give values for inter–particle distances. Furthermore it does not find any difference between "Cluster 4" and "Cluster 1, 2 and 3" which in reality are slightly different (22 clusters are created in the three first images, and 26 for the fourth one). Usually it is used to test if two images are statistically equivalent.

Thus for the analysis of the real image we choose to use the two methods and the inter–cluster distance will be searched with the neighbors method.

5 Real Al/SiC image analysis

5.1 Inter–particle distance

In order to have a comparison between the real image and random ones, 20 random images are created with the same particles as on the real image but localized at random positions.

Fig. 17 shows the results of the comparison. A first characteristic distance appears at 1 pixel. This is due to the fact that many particles are not separated on the image and we have artificially separated them, see Yotte et al (2001). For so closed particles the distance is set to one pixel. The series [2-3-4-5-6] have their frequencies under the lower limit. Then less particles have their first neighbor laying between 2 and 6 pixels distance than in a random image. It means that the distance before 2 (first number of the series) is a maximal intra–cluster inter–particle distance. Thus a first structure exists on the image with these very closed particles (at 1 pixel distance from each other). Similarly distances 8 and 12-13 are characteristic with frequency value under the lower limit.

Fig. 18 shows the result of the analysis through the F function. As for the neighbor frequencies, the 1 pixel distance is characteristic because the value is over the upper limit, but all the other characteristic distances are characteristic because their frequency lays under the lower limit. We use the same method to identify characteristic values: a series of values is said to be characteristic and a single value is a characteristic distance only if it appears in at least two directions. Thus here the important distances are: 1, [2-3-4], 6, [10-11-12], [13-14], [15-16] and [24-25-26].

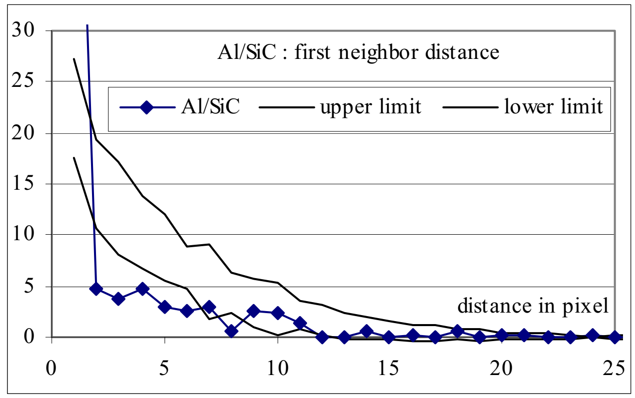
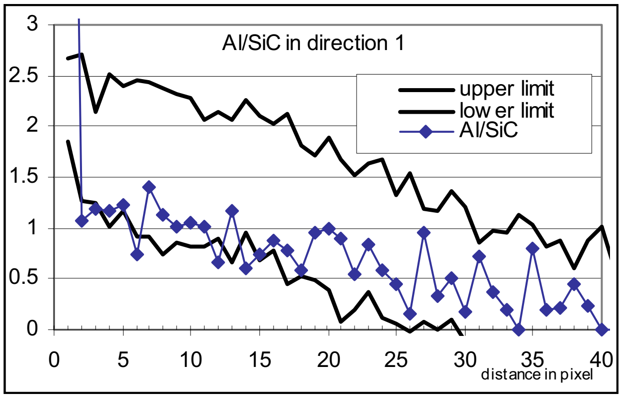


Figure 17 : First neighbor distance of Al/SiC image.

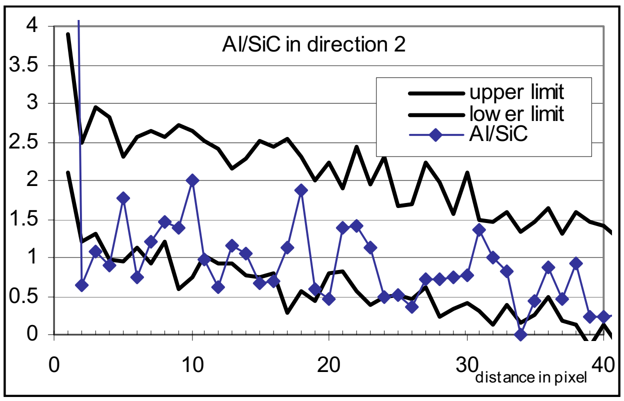
The one pixel distance appears with a high frequency in the two methods. Thus it is related to a first structure. The [2-3-4] series is not characteristic of clusters because their F value in the random method is higher than that of real image, this means only that the value before the series is particular. Indeed the number of particles having a first neighbor at 1 pixel distance is very high. Thus it remains fewer particles than in the random image having a first neighbor at more than 1 pixel distance. This is why the F values for distance higher than 1 are very low. The next characteristic distance is 6. Clusters forming a second structure could have their maximal intra–cluster inter–particle distance at this value. It is possible to observe this second characteristic distance on the second neighbor plot on Fig. 19: here only 1, [6-7-8], [10-11-12-13] and 15 are characteristics. As [6-7-8] are under the lower limit, it means that fewer particles have their second neighbor at this distance thus 5 pixel value is a maximal intra–cluster inter–particle distance for the second structure.

As a conclusion, we observe a first structure whose particles are laying at 1 pixel distance from each other, and a second having an inter–particle distance of 6 (given by F function) or 7 (given by the first neighbor) or 5 (given by the second neighbor). The value of 6 is assumed to be the inter–particle distance of the second structure.

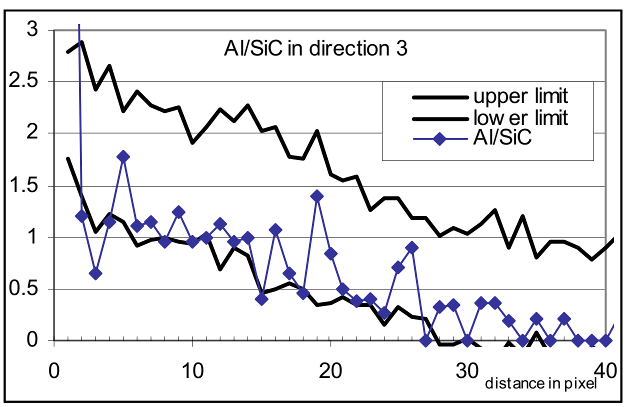
Fig. 20 and 21 show the result of an dilation of respectively 1 pixel and 6 pixels which allowed particle to join up in clusters. The particles in the clusters have their first neighbor laying at a distance between 1 (resp. 6) pixel and twice this value because here all the particles are dilating at the same time. The erosion separates some of



18 (a): Function F applied to image Al/SiC in direction 1



18 (b): Function F applied to image Al/SiC in direction 2



18 (c): Function F applied to image Al/SiC in direction 3

Figure 18 : Results in the three directions of the grid for image 5 analyzed with function F

the particles.

On the two figures isolated particles remain, which have been already seen in Yotte et al (2001). The two struc-

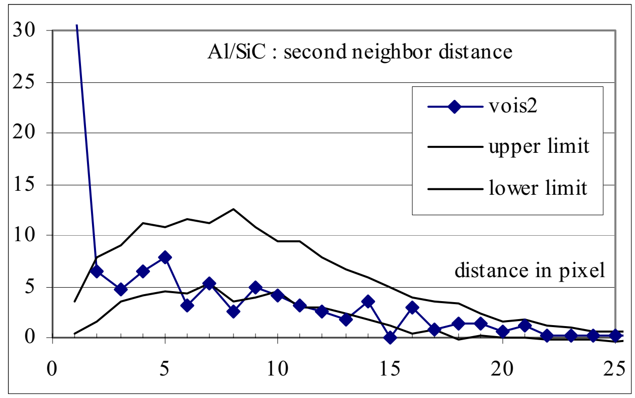


Figure 19 : Second neighbor distance for the Al/SiC image.

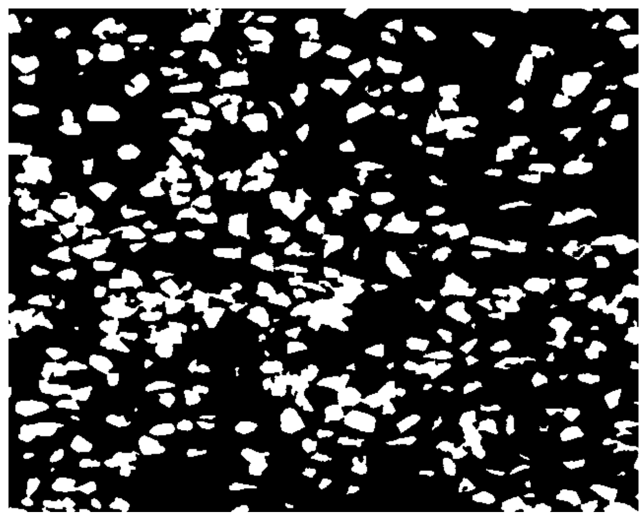


Figure 20 : Al/SiC image after a size 1 dilation and erosion

tures appear clearly on the figures: first structure of small clusters and second one with few clusters of irregular shape and size.

These two constructions enable the inter–cluster distance calculation, which will be done in the next paragraph with the first neighbors distance.

5.2 Inter–cluster distances

Fig. 22 shows the result of the first neighbor method applied to Fig. 20 image. The characteristic distance values are 1, 2 which give frequencies under the lower value, and 5, 7, 9, 10, and 18 which give frequencies over the

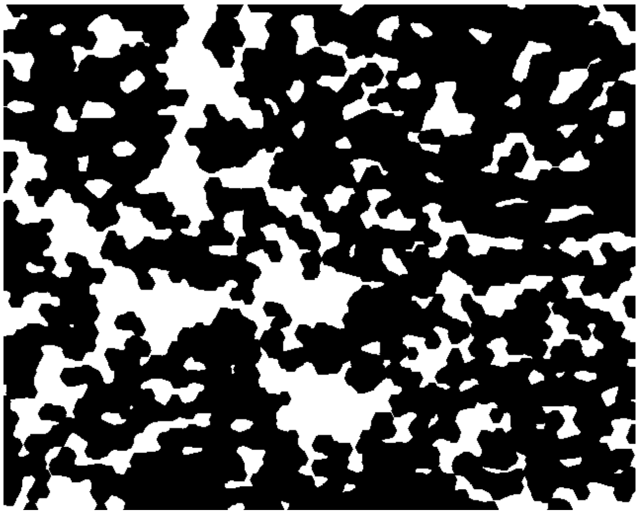


Figure 21 : Al/SiC image after a size 6 dilation and erosion

upper limit. The cluster building operation would have left only few objects at a distance less than 1 or 2 pixels. The result here shows that the dilation/erosion builds the clusters but it modifies the object shape and thus the distances. Nevertheless, the frequencies for 1 and 2 pixel distances are very low. The next characteristic distances are 5 and 7 pixels, which are in agreement with the 6 and 8 distance for the particles. Similarly the 9-10 distance seems in agreement with the 12-13 result for particles.

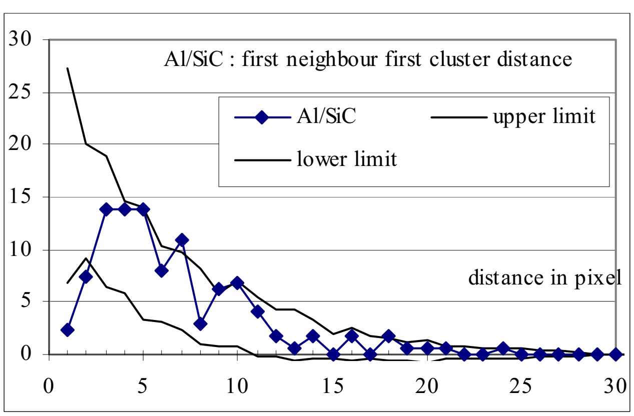


Figure 22 : Distance of the small clusters to their first neighbors

The difference between the particle and this result comes from the comparison which is not done with the same random images: in the case of clustered images there are less objects to cover the same surface. This explains why in the case of particle distance these values have frequencies under the lower limit and in this case they have frequencies over the upper limit. Nevertheless these distances are characteristics of another structure which appears in Fig. 21.

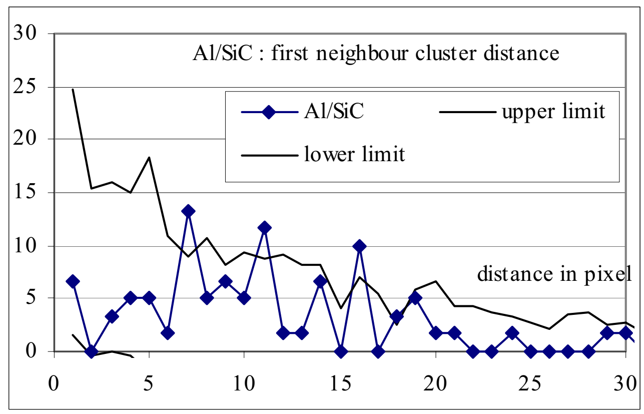


Figure 23 : Distance of the large clusters to their first neighbors

Fig. 22 shows the distances to their first neighbors of the second size clusters, characterized by a intra–cluster inter–particle distance of 6 or 7 pixels. Distances from 1 to 6 show small frequencies (equivalent to 0 to 4 objects), which are due to the cluster building process. Some distances are characteristics: 7, 11, 16, 18 and 19 pixels. Fast all the clusters lay at distances ranging from 7 to 20 pixels. The same trend appears on Fig. 21 curves.

6 Conclusion

Three methods for identifying inter–particle distances have been tested in this work: covariance, distance to neighbors and a function characteristic of distances will call F.

Covariance enables the differentiation of the images but do not gives the distances characteristic of an image.

Neighbor method and F function gives characteristic distance values, which are not always the same but the cross–checking of the two set of distance gives the intra–cluster minimal and maximal inter–particle distance.

We applied these two last methods on an Al/SiC image and evaluated the inter–particle distances and inter–

cluster distances. This underlines the presence of two structures in this image.

This work, together with a previous one on the particle distribution, will enable the construction of images which have the same characteristics as the real one.

These methods are not limited to composite materials. It can be used for any heterogeneous material, for which inter component distance is important for some behavior.

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