

Unit-Cell Model of 2/2-Twill Woven Fabric Composites for Multi-Scale Analysis

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Abstract: A micromechanical unit-cell model was developed for 2/2-twill woven fabric composites so that the model could be implemented for the multi-scale micro/macro-mechanical analysis of 2/2-twill composite structures. The unit-cell model can compute effective material properties of a 2/2-twill composite and decompose the effective stresses (strains) of the woven fabric composite into the stresses (strains) of the tows. When this unit-cell module is incorporated into the multi-scale analysis by combining with other modules developed previously, the residual strength and stiffness of a laminated structure made of 2/2-twill woven fabric composites can be predicted along with damage progression in the structure. Damage is described at the basic material units of the composite structure, the fibers and matrix. Comparing the predicted results to other available data in open literature validated the developed unit-cell model and the multi-scale analysis.

keyword: woven fabric composite, unit-cell, multi-scale, 2/2-twill composite

1 Introduction

There is an ever-increasing need for lightweight, strong, inexpensive materials in all facets of life. These materials are being used in the automotive industry to improve fuel efficiency, in the military service for lightweight weaponry and body armor, and in the aerospace community to reduce payload weight.

The mechanical properties of continuous fiber composite strands are directional. In traditional continuous fiber laminated composites, the fibers are aligned parallel to the plane direction providing very desirable in-plane mechanical properties, but little in the transverse plane mechanical properties.

Alternatively woven fabric composites are being developed which provide an increase in mechanical properties in both the in-plane and transverse plane directions. To take full advantage of these composites requires accurate prediction of the effective properties of stiffness and strength (Michel et. al., 2000; Kailasam et al., 2000).

There has been extensive research in the area of woven fabric composites, and some of them related to the present study are discussed here. Ishikawa and Chou (1983) conducted some of the earliest research and proposed three models for estimating properties of elastic materials. Their first model, the mosaic model, bounded the elastic modulus by using various assumptions. However, this model did not account for fiber undulation. Their second model, the fiber undulation model, addressed the undulation in continuous fiber composite. The primary drawback of these two methods was that they were single dimensional analysis and could not account for interactions between the cross-ply. This led to their third model, the bridging model, which addressed the interaction between the undulation region and its surroundings.

Zhang and Harding (1990) suggested an approach using a strain energy method coupled with finite element analysis. The principle drawback was that undulation was accounted for in only one direction. Naik and Shembekar (1992) expanded the fiber undulation model to include undulation in two directions. Later Naik and Ganesh (1992) suggested the slice array model and the element array model. In both these models, the unit cell was sliced and the elastic property in each slice was recombined to determine the overall material property.

Cox *et al.* (1994), Thompson (1993), and Whitcomb and Srirangan (1996) suggested three-dimensional finite element models. The major drawback of these analyses is the large computational cost of doing the analysis on real structures. Most of those studies considered the plain weave composite and none of them examined the 2/2-twill woven fabric composite. However, different

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weaving patterns result in very different effective material properties so that it is necessary to develop a model for the 2/2-twill composite for its own application.

A few studies considered 2/2-twill composites (Scida et al. (1999); Chaphalkar and Kelkar (2001); Ng et al. (1998)). Those computed the effective modulus but did not compute the stresses and strains at the constituent material level. The constituent level stresses and strains were computed in Refs. (Bohm et al.; Raghavan and Ghosh(2004)) for particulate and fibrous composites but not for woven fabric composites.

A small number of studies have considered multi-scale approaches connecting the macro-level to the micro-level bi-directionally. Some of the studies decomposed laminate strains and stress into the fiber and matrix level stresses to predict failure or damage of constituent materials. In particular, the work of Kwon and his co-workers (1993, 1995, 2001, 2002) used bi-directional unit cell models. This model is thought to be more efficient than the model of Garnich and Hansen (1996), which suggested a bi-directional model that incorporated a finite element analysis to compute the effective composite material properties from constituents' properties. Other models, such as Pechold and Rahman (1994), used the simple strength of material approach, neglecting Poisson's effect.

The objective of this study was to develop an efficient and accurate unit-cell module for 2/2-twill woven fabric composites that could be implemented into a multi-level, multi-scale analysis technique predicting effective stiffness and strength along with progressive damages in composite structures. All the damages were described at the basic material units like the fiber and matrix materials. The rest of the paper is organized as follows: description of the analysis technique, results and discussion of examples that validate the analysis technique, and conclusions.

2 Multi-Scale Micro/Macro-mechanical Analysis

Kwon and his co-workers (1993, 1995, 2001, 2002) developed the basic framework of the multi-scale micro/macro-mechanical model previously. This multi-scale approach was applied to particulate composites, continuous fiber composites, and plain weave composites in previous works. The approach is extended to 2/2-twill woven fabric composites in this study.

The multi-scale micro/macro-mechanical model consists of three interconnected modules as shown in Figure 1. The three modules are the *fiber-strand module*, *strand-fabric module*, and *lamination module*. Using these modules, progressive damage in a laminated 2/2-twill woven fabric composite structure can be described at the fiber and matrix material level, the basic units of the structure. Damage can be classified as matrix damage, fiber breakage, and interface debonding. The effects of damage can be simulated at the composite structural level. In other words, residual strength and stiffness of a composite structure can be predicted based on the progressive damage at the fiber and matrix level.

The overall procedure of the multi-scale micro/macro-analysis is explained below for analyses of laminated 2/2-twill woven fabric composite structures.

1. Compute effective material properties of tows in the matrix (called unidirectional strand henceforth) using the fiber-strand module from the material properties of fibers, matrix, and their volume fractions.
2. Compute effective properties of the 2/2-twill woven fabric lamina from the material properties of the unidirectional strand and their geometric data of weaving using the strand-fabric module.
3. Calculate effective properties of a laminated woven fabric composite from the property of each lamina and orientation using the lamination module.
4. Apply a load to the structure and determine the deformation and strains (stresses) from the finite element analysis using the material properties obtained in the previous step.
5. Compute the strains (stresses) at each lamina from the deformation determined at the laminated structural analysis using the lamination module.
6. Decompose the strains (stresses) at a lamina into the strains (stresses) at the unidirectional strands using the strand-fabric module.
7. Decompose the strains (stresses) at the unidirectional strand into the strains (stresses) in the fiber and the matrix using the fiber-strand module.
8. Apply damage (failure) criteria to determine any initiation of new damage or progression of existing damage at the fiber and matrix level.

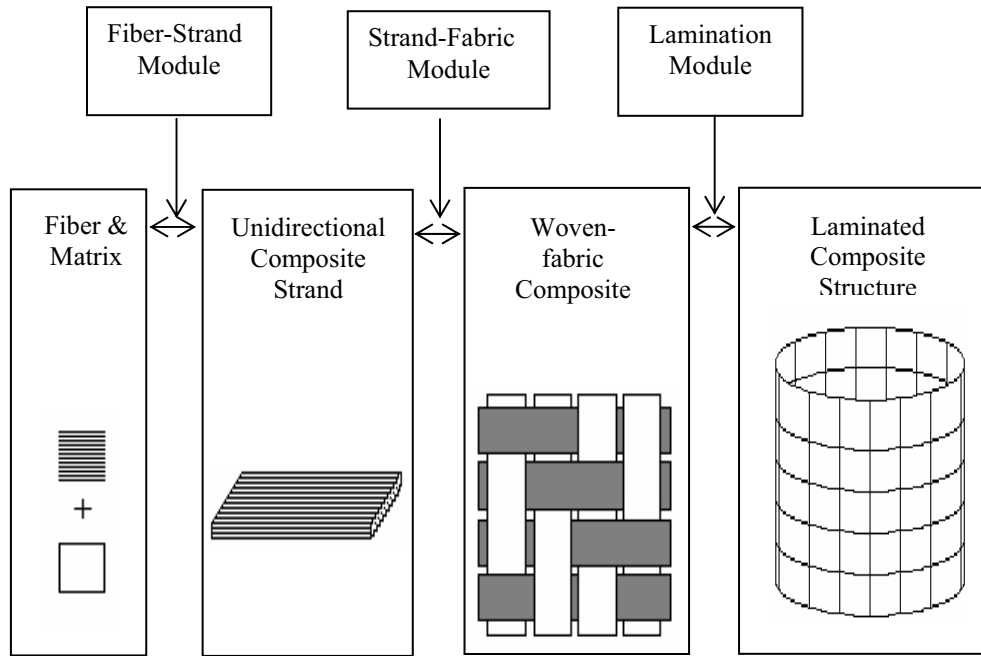


Figure 1 : Schematic of Multi-Level and Multi-Scale Analysis

9. Once there is new damage or progression of damage at the fiber and matrix level, the degraded material properties of the unidirectional strand are computed depending on the damage mode and state using the fiber-strand module.
10. Continue steps 2 - 9 until the ultimate failure of the composite structure occurs.

As explained in the analysis procedure, each module has bi-directional passage to complete the multi-scale analysis. Each module is described below. However, because some of the modules were discussed in detail in previous work by Kwon and his co-workers (1993, 1995, 2001, 2002), those are explained briefly here.

2.1 Fiber-Strand Module

The fiber-strand module relates the constituent materials, e.g. the fiber and matrix, to the unidirectional composite strand. This module serves to determine the effective material properties of the strand from those of the constituent materials. In addition, it also decomposes the stresses and strains of the strand into fiber/matrix stresses and strains so that damage criteria can be applied to the fiber/matrix level.

This module was developed using a unit-cell with four sub-cells. One of the sub-cells represents the fiber and the other three denote the matrix. Within each sub-cell, stresses and strains are assumed to be uniform. In order to develop the mathematical equations of the fiber-strand module, the following equations are solved together: equilibrium at the interfaces of sub-cell, deformation compatibility among sub-cells, the volume average of sub-cells, and the constitutive equations of each sub-cell. Then, the resultant equations are summarized below:

$$[E^{str}] = [V][E][R_2] \quad (1)$$

$$\{\epsilon^\alpha\} = [R_2]\{\epsilon^{str}\} \quad (2)$$

where $[E^{str}]$ is the effective material property matrix for the composite strand, $[V]$ is the matrix containing volume fractions of each sub-cell, $[E]$ is the matrix of material properties of the constituents, and $[R_2]$ is the decomposition matrix that transforms strand strains $\{\epsilon^{str}\}$ into the fiber and matrix strains $\{\epsilon^\alpha\}$. Detailed development of these two expressions, which provide the bi-directional passage, was discussed in previous studies (Kwon (1993); Kwon and Berner (1995)).

Once fiber and matrix stresses are computed, failure criteria are applied to the stresses. For the fiber breakage criterion, the normal stress along the fiber orientation does not exceed the strength of the fiber material. Usually, different strength values are used for tensile and compressive loads, respectively, in consideration of fiber buckling. The matrix material uses an isotropic failure criterion like the maximum shear stress criterion as long as the material is considered as isotropic. If there is failure in fiber and/or matrix material(s), the material properties like elastic modulus are degraded so as to include the failure effects. Then, the degraded material properties are used in the fiber-strand module to compute the effective properties of the locally failed strands.

2.2 Strand-Fabric Module for 2/2 Twill Composites

The strand-fabric module relates effective material properties of a unidirectional composite strand to those of a 2/2-twill woven fabric. The module also decomposes the woven fabric stresses and strains into strand stresses and strains. In order to perform these functions, a unit-cell model is developed as described below.

Figure 2 represents a 2/2-twill fabric repeating unit-cell. The unit-cell is subdivided into 77 sub-cells. Among the 77 sub-cells, some sub-cells have the same state of stresses. As a result, there are 17 independent sub-cells. Figure 3 is a graphical representation of a 2/2-twill unit-cell showing the 17 uniquely stressed sub-cells. Sub-cells with matching letter-number designations are assumed to have the same stress states. Four have fibers in the 1-direction, four have fibers in the 2-direction, four have fibers that are inclined to the 1-2 plane, and five are filled with the matrix material. The following equations are shown for normal components of stresses and strains. Very similar expressions can be written for the shear stress and strain components to complete the derivation. However, the latter expressions are omitted to save space.

The first set of equations is equilibrium at interfaces of all sub-cells. The average stress and strain of each sub-cell are used in all the subsequent equations. Applying equilibrium to any two neighboring sub-cells results in the following normal stress equations, where the subscripts indicate stress components and the superscripts designate

the sub-cell numbers.

$$\begin{aligned} \sigma_{11}^2 &= \sigma_{11}^8; & \sigma_{11}^2 &= \sigma_{11}^{15}; & \sigma_{11}^2 &= \sigma_{11}^{14}; & \sigma_{11}^3 &= \sigma_{11}^7; \\ \sigma_{11}^3 &= \sigma_{11}^{15}; & \sigma_{11}^3 &= \sigma_{11}^{14}; & \sigma_{11}^5 &= \sigma_{11}^9; & \sigma_{11}^4 &= \sigma_{11}^6; \\ \sigma_{11}^1 &= \sigma_{11}^{16}; & \sigma_{11}^1 &= \sigma_{11}^{17}; & \sigma_{11}^1 &= \sigma_{11}^{10} + \sigma_{11}^{11}; \\ \sigma_{11}^1 &= \sigma_{11}^{12} + \sigma_{11}^{13}; \end{aligned} \quad (3)$$

$$\begin{aligned} \sigma_{22}^4 &= \sigma_{22}^{16}; & \sigma_{22}^4 &= \sigma_{22}^{17}; & \sigma_{22}^4 &= \sigma_{22}^{12}; & \sigma_{22}^5 &= \sigma_{22}^{17}; \\ \sigma_{22}^5 &= \sigma_{22}^{17}; & \sigma_{22}^5 &= \sigma_{22}^{11}; & \sigma_{22}^3 &= \sigma_{22}^{13}; & \sigma_{22}^2 &= \sigma_{22}^{10}; \\ \sigma_{22}^1 &= \sigma_{22}^{14}; & \sigma_{22}^1 &= \sigma_{22}^{15}; & \sigma_{22}^1 &= \sigma_{22}^6 + \sigma_{22}^7; \\ \sigma_{22}^1 &= \sigma_{22}^8 + \sigma_{22}^9; \end{aligned} \quad (4)$$

$$\begin{aligned} \sigma_{33}^2 &= \sigma_{33}^5; & \sigma_{33}^3 &= \sigma_{33}^4; & \sigma_{33}^6 &= \sigma_{33}^7; \\ \sigma_{33}^8 &= \sigma_{33}^9; & \sigma_{33}^{10} &= \sigma_{33}^{11}; & \sigma_{33}^{12} &= \sigma_{33}^{13}; \end{aligned} \quad (5)$$

Eq. (1) is a set of normal stress equilibrium in the 1-direction while Eqs. (2) and (3) are the equilibrium in the 2- and 3-direction, respectively.

Another set of equations is strain compatibility. Assuming uniform deformation of the unit-cell supplies the following equations.

$$\begin{aligned} 2a(\epsilon_{11}^2 + \epsilon_{11}^3) + h(\epsilon_{11}^7 + \epsilon_{11}^8) &= \\ 2a(\epsilon_{11}^4 + \epsilon_{11}^5) + h(\epsilon_{11}^6 + \epsilon_{11}^9) & \end{aligned} \quad (6)$$

$$\begin{aligned} 2a(\epsilon_{11}^2 + \epsilon_{11}^3) + h(\epsilon_{11}^8) &= \\ 2a(\epsilon_{11}^4 + \epsilon_{11}^5) + h(\epsilon_{11}^9) & \end{aligned} \quad (7)$$

$$\begin{aligned} 2a(\epsilon_{11}^2 + \epsilon_{11}^3) + h(\epsilon_{11}^8 + \epsilon_{11}^{14} + \epsilon_{11}^{15}) &= \\ a(\epsilon_{11}^{10} + \epsilon_{11}^{13} + \epsilon_{11}^{16} + \epsilon_{11}^{17}) + 3h(\epsilon_{11}^1) & \end{aligned} \quad (8)$$

$$\epsilon_{11}^{10} + \epsilon_{11}^{12} = \epsilon_{11}^{11} + \epsilon_{11}^{13} \quad (9)$$

$$\begin{aligned} 2a(\epsilon_{22}^2 + \epsilon_{22}^3) + h(\epsilon_{22}^{11} + \epsilon_{22}^{12}) &= \\ 2a(\epsilon_{22}^4 + \epsilon_{22}^5) + h(\epsilon_{22}^{10} + \epsilon_{22}^{13}) & \end{aligned} \quad (10)$$

$$\begin{aligned} 2a(\epsilon_{22}^2 + \epsilon_{22}^3) + h(\epsilon_{22}^{13}) &= \\ 2a(\epsilon_{22}^4 + \epsilon_{22}^5) + h(\epsilon_{22}^{12}) & \end{aligned} \quad (11)$$

$$\begin{aligned} 2a(\epsilon_{22}^4 + \epsilon_{22}^5) + h(\epsilon_{22}^{12} + \epsilon_{22}^{16} + \epsilon_{22}^{17}) &= \\ a(\epsilon_{22}^6 + \epsilon_{22}^9 + \epsilon_{22}^{14} + \epsilon_{22}^{15}) + 3h(\epsilon_{22}^1) & \end{aligned} \quad (12)$$

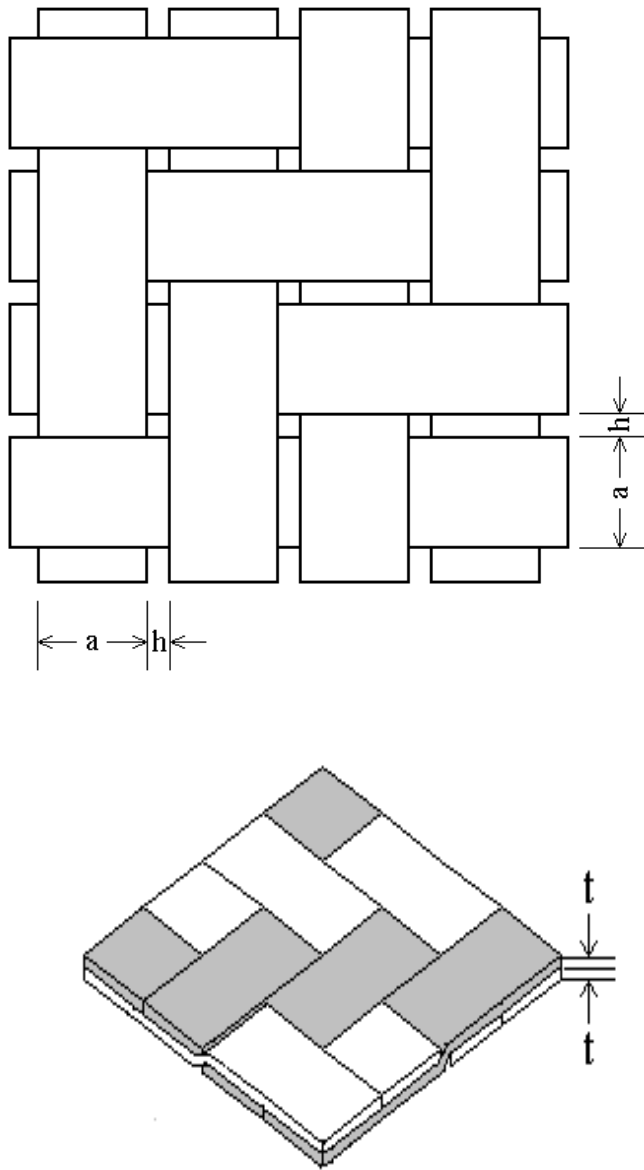


Figure 2 : Unit-Cell for 2/2-Twill Composite

$$\epsilon_{22}^6 + \epsilon_{22}^8 = \epsilon_{22}^7 + \epsilon_{22}^9 \quad (13)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{16} \quad (14)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{15} \quad (15)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{17} \quad (16)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{14} \quad (17)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^1 \quad (18)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^6 + \epsilon_{33}^7 \quad (19)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{12} + \epsilon_{33}^{13} \quad (20)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^6 + \epsilon_{33}^8 \quad (21)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^{10} + \epsilon_{33}^{11} \quad (22)$$

$$\epsilon_{33}^2 + \epsilon_{33}^5 = \epsilon_{33}^3 + \epsilon_{33}^4 \quad (23)$$

Here a and h are the dimensions of the fill and warp strands of the composite material as shown in Fig. 2.

The next set of equations is the constitutive equation for every sub-cell as given below:

$$(\sigma_{ij}^{str})^n = (E_{ijkl}^{str})^n (\epsilon_{kl}^{str})^n \quad (24)$$

where $(\sigma_{ij}^{str})^n$ and $(\epsilon_{kl}^{str})^n$ are the n^{th} sub-cell stresses and strains, and $(E_{ijkl}^{str})^n$ is the sub-cell material property matrix in terms of the unit-cell axes. The material property matrix is determined from proper transformation as shown below:

$$[E_{ijkl}^{str}] = [T_\sigma]^{-1} [\bar{E}^{str}] [T_\epsilon] \quad (25)$$

in which $[T_\sigma]$ and $[T_\epsilon]$ are the stress and strain transformation matrices (Kwon and Bang, 2000), and $[\bar{E}^{str}]$ is the material property matrix based on the strand orientation of the sub-cell.

The final set of equations is the relationship between the woven fabric unit-cell stresses (strains) and the strand sub-cell stresses (strains). The woven fabric unit-cell stresses and strains are computed as volumetric average of sub-cell stresses and strains.

$$\sigma_{ij}^{wf} = \sum_{n=1}^{17} V^n (\sigma_{ij}^{str})^n \quad (26)$$

$$\epsilon_{ij}^{wf} = \sum_{n=1}^{17} V^n (\epsilon_{ij}^{str})^n \quad (27)$$

where V^n is the volume fraction of the n^{th} sub-cell. After solving the algebraic equations given above, the resulting relationships are formed:

(14) solving the algebraic equations given above, the resulting relationships are formed:

$$E_{ijkl}^{wf} = f((E_{ijkl}^{str})^n, a, h, t) \quad (28)$$

$$(16) \quad (17)$$

$$(17) \quad (18)$$

$$(18) \quad (\epsilon_{ij}^{str})^n = g(\epsilon_{ij}^{wf}, a, h, t) \quad (29)$$

(19) These two equations provide the bi-directional passage of the strand-fabric module. Eq. (28) is used to compute the

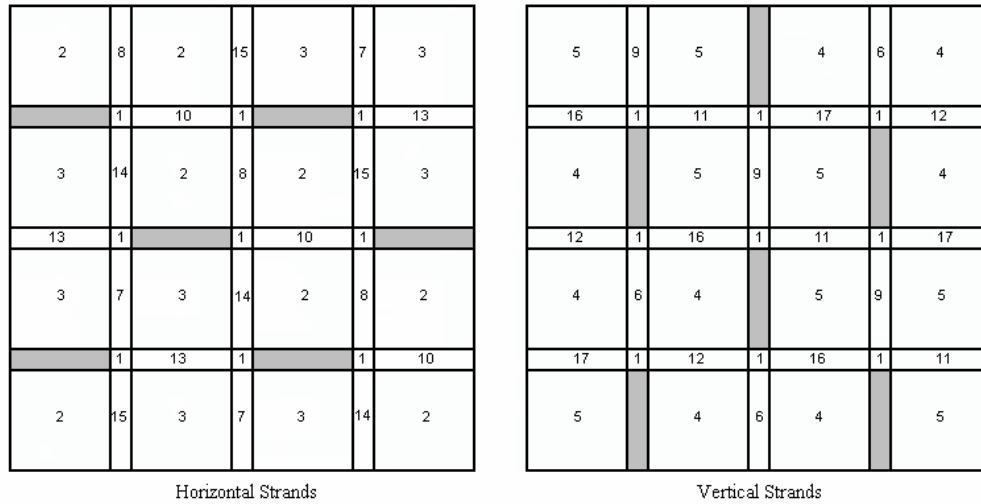


Figure 3 : Sub-cell Numbering

effective material properties, E_{ijkl}^{wf} , of a 2/2-twill woven fabric composite from the material properties, $(E_{ijkl}^{str})^n$ of unidirectional strand and the weaving geometric dimensions, a , h , and t in Fig. 2. On the other hand, Eq. (29) decomposes the 2/2-twill composite strains into the sub-cell strains. Then, sub-cell stresses are calculated using Eq. (24).

2.3 Lamination Module

The lamination module relates the effective material properties of a woven fabric lamina to the laminated properties of multiple layers. These properties are used in the finite element analysis of a laminated structure made of a 2/2-twill woven fabric composite. Once the deformations, stresses, and strains are computed for the laminated composite structure, the module computes the stresses and strains of each lamina of the composite structure. In order to undertake these functions, the lamination theory is used. Either the classical lamination theory or a higher order theory may be used in the module. In this study, the classical theory was used. It is available in most textbooks, thus it was not presented here.

3 Results and Discussion

The developed model was validated by comparing its results with the published results of other 2D and 3D micro-mechanics models for S2 glass/C-50 resin, E-glass/epoxy, and carbon fiber/Bakelite composites. The material properties of the constituent fibers and matri-

ces are tabulated in Table 1 while those of unidirectional strands are tabulated in Table 2. The predicted material properties of different 2/2-twill woven fabric composites are compared to the experimental and analytical results in Tables 3 through 5. The present model resulted in quite accurate effective material properties as shown in the tables.

The next study investigates the progressive failure of a composite plate under an inplane tensile load. As expected, the first type of failure in a woven fabric composite is cracking of the matrix material normal to the load direction, called transverse cracking. Thus, matrix material in the normal direction to the load was degraded gradually and the reduced effective material properties were computed. Figure 4 is a plot of the normalized reduced effective modulus in the transverse direction (E_y) as a function of the progressive transverse cracking in the matrix material (i.e. percentage reduction in the matrix modulus in the transverse direction). The reduced modulus was normalized with respect to the virgin modulus. The plot indicates the transverse elastic modulus decreases nonlinearly along with the percentage matrix degradation. The transverse modulus is minimally affected until the matrix damage is more than 50 percent. As the percentage matrix damage becomes very high, the transverse modulus drops significantly as seen in the figure. The present prediction closely parallels that in Ref. (Chaphalkar and Kelkar, 2001).

The final study simulated failure initiation and progres-

Table 1 : Material Properties of the Constituent Materials

	E_1 (GPa)	E_2, E_3 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}	σ_{ten} (MPa)
S2 Glass	221.0	13.8	13.8	5.5	0.25	0.25	3585
C-50 resin	4.4	4.4	1.6	1.6	0.34	0.34	159
E-glass	73	73	30.4	30.4	0.2	0.2	2500
Epoxy	3.2	3.2	1.16	1.16	.38	.38	70
Carbon	230	24	50	33	.28	.25	3240

Table 2 : Properties of the Composite Strands

	E_1 (GPa)	E_2, E_3 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}
S2/C-50	71.6	23.7	10.2	8.4	.22	.27
E-glass/Epoxy	55.7	18.5	6.89	6.04	.22	.34
Carbon/Bakelite	137	9.57	4.74	3.23	.31	.45

Table 3 : Properties of the S2/C50 Woven Fabric

	$E_x=E_y$ (GPa)
Experiment*	28.7
Analytical*	30.6
Present	28.5

*Chaphalkar and Kelkar 1999

Table 4 : Properties of the E-Glass/Epoxy Woven Fabric

	$E_x=E_y$ (GPa)	E_z (GPa)
Experiment*	19.24	N/A
Analytical*	19.54	10.93
Present	19.67	9.21

*Scida et. al. 1999

Table 5 : Properties of the Carbon/Epoxy Woven Fabric

	$E_x=E_y$ (GPa)	E_z (GPa)
Experiment*	49.38	N/A
Analytical*	46.11	8.18
Present	50.3	7.52

*Scida et al. 1997

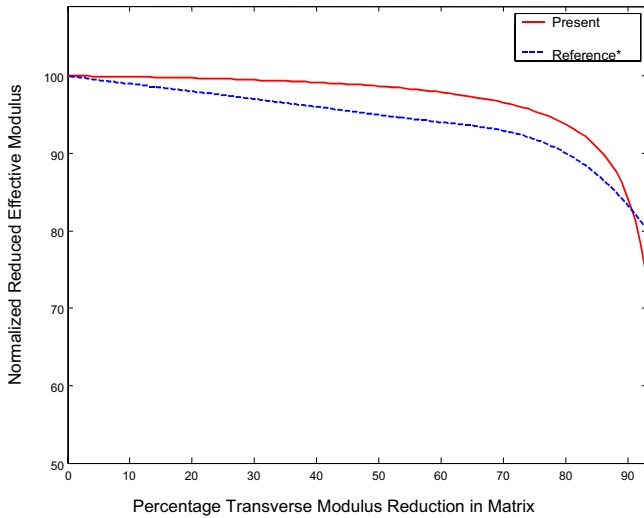


Figure 4 : Reduction of Effective Transverse Modulus as a Function of Transverse Matrix Damage (*Chaphalkar and Kelkar, 2001)

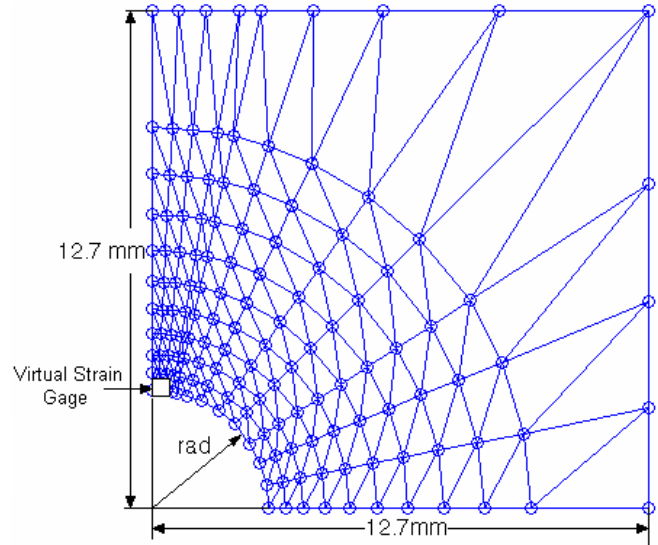


Figure 5 : Finite Element Mesh of a Quarter of the Plate with a Hole

sion until complete rupture of 2/2-twill woven fabric composite plates with drilled holes at the center. Plates with three different hole sizes were considered, respectively. The hole diameters were 3 mm, 6 mm, and 9 mm. Figure 5 shows a finite element mesh of a quarter of a plate. Symmetric boundary conditions were applied to the bottom and left boundaries, and a uniform displacement was applied to the right boundary of the mesh. The displacement was applied incrementally to the specimens. The analysis resulted in a stress versus strain plot for a given hole diameter (3 mm). The present stress-strain curve around the notch tip was plotted in Fig. 6 and compared to the results in Ref. (Scida et al.1997). Both results compared very well.

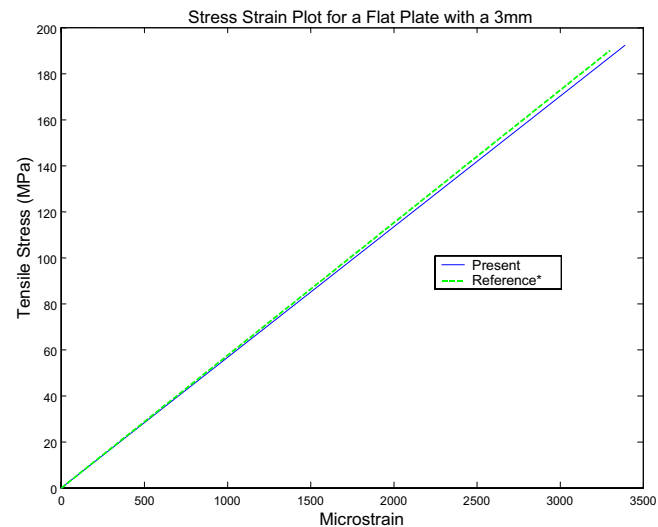


Figure 6 : Stress-Strain Curve at the Notch Tip Area of a Drilled Plate (* Ng et al. 1998b)

In this study, ultimate failure strengths of the three perforated composite plates were predicted using the present analysis model and compared to the experimental data obtained by Ng *et al.* (1998b). The results are tabulated in Table 6. The presented predicted strengths are close to both experimental and previously predicted results.

4 Conclusions

The multi-level and multi-scale model presented here provides a computationally efficient tool for analysis of a laminated composite structure made of 2/2-twill woven fabric composites. The analysis model was based

on three independent modules called fiber-strand, strand-fabric, and lamination modules. Each module communicates with another module bi-directionally. Additionally, each one can be used separately for its own purposes. This analysis tool was accurate in predicting effective composite material properties from the constituent fiber and matrix properties as demonstrated in three different composite material cases. It was also used to predict pro-

Table 6 : Predicted and Experimental Strengths of Plate with Center Drilled Hole

Hole Diameter (mm)	Experimental Strength (MPa)*	Predicted Strength (MPa)*	Present Predicted Strength (MPa)
3	435	499	494
6	395	460	438
9	333	377	354

* Ng et al. (1998b)

gressive damage in a composite structure. The damage was described at the basic material units of the composites, i.e. the fiber and matrix materials. Along with the progressive damage, reduced effective stiffness and strength of a laminated composite structure could be determined. The examples presented in this paper showed that the present analysis tool provided accurate data in predicting effective stiffness and strength.

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