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Sensitivity of the skin tissue on the activity of external heat sources

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Abstract: In the paper the analysis of transient temperature field in the domain of biological tissue subjected to an external heat source is presented. Because of the geometrical features of the skin the heat exchange in domain considered is assumed to be one-dimensional. The thermophysical parameters of successive skin layers (dermis, epidermis and sub-cutaneous region) are different, at the same time in sub-domains of dermis and sub-cutaneous region the internal heat sources resulting from blood perfusion are taken into account. The degree of the skin burn results from the value of the so-called Henriques integrals. The first and the second order sensitivity of these integrals with respect to the thermophysical parameters are analyzed. On the stage of numerical computations the boundary element method has been used. In the final part of the paper the results of computations are shown.

keyword: bioheat transfer, burns prediction, sensitivity analysis, boundary element method

1 Introduction

The susceptibility of human body to influence of external thermal interactions is very discriminated. It is caused, first of all, by the different values of thermphysical parameters of the skin - they are conditioned by the sex, age and even the profession of concrete person. The same external heat flux and the same its exposure time can give quite different effects (e.g. the degree of burn) according to the individual features.

It seems that the methods of sensitivity analysis can be in such case very effective, because they give the information concerning the mutual connections between the thermal processes proceeding in the domain analyzed and the thermal effects (in this case the damage of the skin) for a large scale of individual thermophysical parameters - in particular if the higher order sensitivity is introduced into considerations.

Thermal damage of skin begins when the temperature at the basal layer (the interface between epidermis and dermis) rises above 44[^{o}C] (317[K]). Henriques (1947) found that the degree of skin damage could be predicted on the basis of the integrals

$$I_b = \int_0^{\tau} P_b(T_b) \exp\left(-\frac{\Delta E}{RT_b(t)}\right) dt \qquad (1)$$

and

$$I_d = \int_0^{\tau} P_d(T_d) \exp\left(-\frac{\Delta E}{RT_d(t)}\right) dt \qquad (2)$$

where $\Delta E/R$ [K] is the ratio of activation energy to universal gas constant, P_b, P_d [1/s] are the pre-exponential factors, while T_b, T_d [K] are the temperature of basal layer (the surface between epidermis and dermis) and dermal base (the surface between dermis and subcutaneous region), [0, τ] is the time interval considered.

First degree burns are said to occur when the value of the burn integral (1) is from the interval $0.53 < I_b \leq 1$, while the second degree burns when $I_b > 1$, see Henriques (1947), Torvi and Dale (1994). The third degree appears when the integral $I_d > 1$. So, in order to determine the values of integrals I_b and I_d the heating and next the cooling curve for the basal surface and dermal base must be known.

2 Mathematical model of thermal processes

We consider the 1D heterogeneous domain in which one can distinguish the following sub-domains: epidermis of

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thickness L_1 [m], dermis of thickness $L_2 - L_1$ and subcutaneous region of thickness $L - L_2$ - Fig. 1. The thermophysical parameters of these regions are equal to λ_e [W/mK] (thermal conductivity) and c_e [J/m^{3}K] (specific heat per unit of volume), e=1, 2, 3.

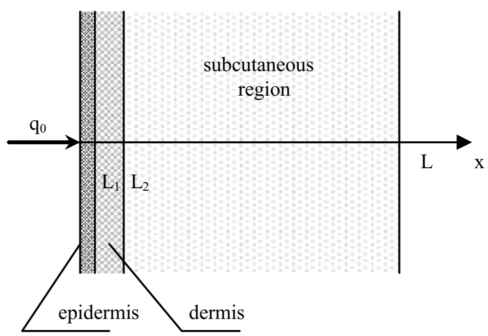

Figure 1 : Domain considered

The transient bio-heat transfer in domain of skin is described by following system of equations, see Pennes (1948), Brinck and Werner (1994), Shaw, Preston and Bacon (1996), Liu and Xu (2000), Majchrzak (1998):

$$x \in \Omega_e : \quad c_e \frac{\partial T_e}{\partial t} = \lambda_e \frac{\partial^2 T_e}{\partial x^2} + k_e (T_b - T_e) + Q_{me} \tag{3}$$

where $k_e = G_e c_B$ is the product of blood perfusion rate and volumetric specific heat of blood, T_B is the blood temperature and Q_{me} is the metabolic heat source. On the contact surfaces between the sub-domains the continuity conditions in the form

$$x \in \Gamma_{1,2} : \quad \begin{cases} q_1 = q_2 = q_b \\ T_1 = T_2 = T_b \end{cases} \tag{4}$$

and

$$x \in \Gamma_{2,3} : \quad \begin{cases} q_2 = q_3 = q_d \\ T_2 = T_3 = T_d \end{cases} \tag{5}$$

are given, where $q_e = -\lambda_e \partial T_e / \partial x$.
On the skin surface the following condition is assumed

$$x \in \Gamma_0 : \quad \begin{cases} q_1 = q_0, & t \le t_0 \\ q_1 = \alpha (T_1 - T^\infty) & t > t_0 \end{cases} \tag{6}$$

where q_0 is the given boundary heat flux, t_0 is the exposure time (see Behnke (1984)), α is the heat transfer coefficient, T^∞ is the ambient temperature. For conventionally assumed boundary Γ_c limiting the system, the no-flux condition can be taken into account. Additionally, for t=0 the initial temperature distribution is known: t=0: $T_e(x,0) = T_p(x)$.

3 The first-order sensitivity analysis

The first order sensitivity analysis of bioheat transfer will be done with respect to the parameters λ_e, c_e, k_e. These parameters we denote by p_s, $s = 1, 2, \ldots, 9$. If the direct method of sensitivity analysis is used (Dems (1986), Davies, Saidel and Harasaki (1997), Majchrzak and Jasiski (2001)), then we should consider nine additional boundary-initial problems resulting from the differentiation of diffusion equations and boundary-initial conditions, this means

$$\begin{cases} x \in \Omega_e : c_e \dfrac{\partial U_{es}}{\partial t} = \lambda_e \dfrac{\partial^2 U_{es}}{\partial x^2} + k_e U_{es} + q_{Ve} \quad e = 1,2,3 \\ x = 0 \ : \begin{cases} V_{1s} = -\dfrac{1}{\lambda_1} \dfrac{\partial \lambda_1}{\partial p_s} q_0 & t \le t_0 \\ V_{1s} = \alpha U_{1s} - \dfrac{1}{\lambda_1} \dfrac{\partial \lambda_1}{\partial p_s} q_1 \, t > t_0 \end{cases} \\ x = L_e : \begin{cases} U_{es} = U_{e+1,s}, & e = 1,2 \\ \dfrac{1}{\lambda_e} \dfrac{\partial \lambda_e}{\partial p_s} q_e + V_{es} = \dfrac{1}{\lambda_{e+1}} \dfrac{\partial \lambda_{e+1}}{\partial p_s} q_{e+1} + V_{e+1,s} \end{cases} \\ x = L \ : V_{3s} = 0 \\ t = 0 \ : U_{es} = 0 \end{cases} \tag{7}$$

where

$$U_{es} = \frac{\partial T_e}{\partial p_s}, \quad V_{es} = -\lambda_e \frac{\partial U_{es}}{\partial x} \tag{8}$$

and

$$q_{Ve} = \left(\frac{c_e}{\lambda_e} \frac{\partial \lambda_e}{\partial p_s} - \frac{\partial c_e}{\partial p_s} \right) \frac{\partial T_e}{\partial t} + \frac{1}{\lambda_e} \frac{\partial \lambda_e}{\partial p_s} \left[k_e T_e - (k_e T_B + Q_{me}) \right] + \frac{\partial k_e}{\partial p_s} (T_B - T_e) \tag{9}$$

Taking into account the form of functionals I_b, I_d the sensitivity of these integrals with respect to the parameters

p_s should be calculated using the formulas

$$\frac{\partial I_b}{\partial p_s} = \int_0^\tau P_b \frac{\Delta E}{R T_b^2(t)} \exp\left[-\frac{\Delta E}{R T_b(t)}\right] U_{bs}(t)\, dt \tag{10}$$

and

$$\frac{\partial I_d}{\partial p_s} = \int_0^\tau P_d \frac{\Delta E}{R T_d^2(t)} \exp\left[-\frac{\Delta E}{R T_d(t)}\right] U_{ds}(t)\, dt \tag{11}$$

where

$$U_{bs}(t) = U_{1s}(L_1, t) = U_{2s}(L_1, t) \tag{12}$$

and

$$U_{ds}(t) = U_{2s}(L_2, t) = U_{3s}(L_2, t) \tag{13}$$

The change of burn integrals connected with the change of parameter p_s results from the Taylor formula limited to the first-order sensitivity, this means

$$I_b(p_s \pm \Delta p_s) = I_b(p_s) \pm U_{bs}\Delta p_s \tag{14}$$

or

$$I_d(p_s \pm \Delta p_s) = I_d(p_s) \pm U_{ds}\Delta p_s \tag{15}$$

The second-order sensitivity analysis

The second order sensitivity analysis requires the differentiation of equations (7) with respect to the parameters p_s. After the mathematical manipulations one obtains

$$\begin{cases} x \in \Omega_e: & c_e \dfrac{\partial W_{es}}{\partial t} = \lambda_e \dfrac{\partial^2 W_{es}}{\partial x^2} + k_e W_{es} + Q_{Ve} \\ x = 0\ : & \begin{cases} Z_{1s} = -\dfrac{1}{\lambda_1}\dfrac{\partial \lambda_1}{\partial p_s}\left(\dfrac{1}{\lambda_1} q_0 - V_{1s}\right), & t \le t_0 \\ Z_{1s} = \alpha W_{1s} - \dfrac{2}{\lambda_1}\dfrac{\partial \lambda_1}{\partial p_s} V_{1s}, & t > t_0 \end{cases} \\ x = L_e: & \begin{cases} W_{es} = W_{e+1,s} \\ \dfrac{2}{\lambda_e}\dfrac{\partial \lambda_e}{\partial p_s} V_{es} + Z_{es} = \dfrac{2}{\lambda_{e+1}}\dfrac{\partial \lambda_{e+1}}{\partial p_s} V_{e+1,s} + Z_{e+1,s} \end{cases} \\ x = L\ : & Z_{3s} = 0, \\ t = 0\ : & W_{es} = 0 \end{cases} \tag{16}$$

where

$$W_{es} = \frac{\partial U_e}{\partial p_s}, \quad Z_{es} = -\lambda_e \frac{\partial W_{es}}{\partial x} \tag{17}$$

and

$$Q_{Ve} = \frac{2}{\lambda_e}\frac{\partial \lambda_e}{\partial p_s}\left[c_e \frac{\partial U_{es}}{\partial t} + k_e U_{es} - \frac{1}{\lambda_e}\left(c_e \frac{\partial T_e}{\partial t} + k_e T_e - (k_e T_B + Q_{me})\right)\right] - \frac{\partial k_e}{\partial p_s} U_{es} \tag{18}$$

The second order sensitivity of burn integrals with respect to the parameters p_s is calculated using the formulas

$$\frac{\partial^2 I_b}{\partial p_s^2} = \int_0^\tau P_b \frac{\Delta E}{R T_b^2(t)}\left[\left(\frac{\Delta E}{R T_b^2(t)} - \frac{2}{T_b(t)}\right) U_{bs}(t) + W_{bs}(t)\right] \cdot \exp\left[-\frac{\Delta E}{R T_b^2(t)}\right] dt \tag{19}$$

and

$$\frac{\partial^2 I_d}{\partial p_s^2} = \int_0^\tau P_d \frac{\Delta E}{R T_d^2(t)}\left[\left(\frac{\Delta E}{R T_d^2(t)} - \frac{2}{T_d(t)}\right) U_{ds}(t) + W_{ds}(t)\right] \cdot \exp\left[-\frac{\Delta E}{R T_d^2(t)}\right] dt \tag{20}$$

where

$$W_{bs}(t) = W_{1s}(L_1, t) = W_{2s}(L_1, t) \tag{21}$$

and

$$W_{ds}(t) = W_{2s}(L_2, t) = W_{3s}(L_2, t) \tag{22}$$

If the second-order sensitivities are taken into account then the formulas (14), (15) can be developed to the more exact form:

$$I_b(p_s \pm \Delta p_s) = I_b(p_s) \pm U_{bs}\Delta p_s + \frac{1}{2} W_{bs}\Delta p_s^2 \tag{23}$$

and

$$I_d(p_s \pm \Delta p_s) = I_d(p_s) \pm U_{ds}\Delta p_s + \frac{1}{2} W_{ds}\Delta p_s^2 \tag{24}$$

4 The method of numerical solution

The basic problem and additional sensitivity problems have been solved using the I scheme of the BEM (Banerjee (1994), Brebbia and Dominguez (1992), Majchrzak (2001)). For mesh-free simulation of diffusion equations see also Golberg and Chen (2001), and for heat conduction in anisotropic bodies Ochiai (2001). The algorithm discussed concerns the following diffusion equation

$$x \in \Omega_e : c_e \frac{\partial F_e(x,t)}{\partial t} = \lambda_e \frac{\partial^2 F_e(x,t)}{\partial x^2} + S_e(x,t), e = 1,2,3 \tag{25}$$

where F_e denotes the temperature or functions resulting from the sensitivity analysis, while $S_e(x,t)$ is the source function. We introduce the time grid $\{t^0, t^1, \ldots\}$ with a constant step $\Delta t = t^f - t^{f-1}$

The I scheme of the BEM for equation (25) and transition $t^{f-1} \to t^f$ leads to the formulas ($e = 1,2,3$)

$$F_e(\xi,,t^f) + \left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f} F_e^*(\xi,,x,t^f,t)\, J_e(x,t)\, dt\right]_{x=L_{e-1}}^{x=L_e}$$
$$= \left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f} J_e^*(\xi,,x,t^f,t)\, F_e(x,t)\, dt\right]_{x=L_{e-1}}^{x=L_e}$$
$$+ \int_{L_{e-1}}^{L_e} F_e^*(\xi,,x,t^f,t^{f-1})\, F_e(x,t^{f-1})\, dx$$
$$+ \frac{1}{c_e}\int_{L_{e-1}}^{L_e} S_e(x,t^{f-1}) \int_{t^{f-1}}^{t^f} F_e^*(\xi,,x,t^f,t)\, dt dx \tag{26}$$

where F_e^* are the fundamental solutions:

$$F_e^*(\xi,x,t^f,t) = \frac{1}{2\sqrt{\pi a_e(t^f-t)}} \exp\left[-\frac{(x-\xi)^2}{4a_e(t^f-t)}\right] \tag{27}$$

where ξ is the point in which the concentrated heat source is applied, $a_e = \lambda_e/c_e$, $J_e(x,t) = -\lambda_e \partial F_e/\partial x$.

The heat fluxes resulting from fundamental solutions are equal to

$$J_e^*(\xi,x,t^f,t) = -\lambda_e \frac{\partial F_e^*(\xi,x,t^f,t)}{\partial x}$$
$$= \frac{\lambda_e(x-\xi)}{4\sqrt{\pi}[a_e(t^f-t)]^{3/2}} \exp\left[-\frac{(x-\xi)^2}{4a_e(t^f-t)}\right] \tag{28}$$

Assuming that for $t \in [t^{f-1}, t^f] : F_e(x,t) = F_e(x,t^f)$ and $J_e(x,t) = J_e(x,t^f)$ one obtains the following form of equations (26)

$$F_e(\xi,t^f) + g_e(\xi,L_e)\, J_e(L_e,t^f)$$
$$- g_e(\xi,L_{e-1})\, J_e(L_{e-1},t^f) = h_e(\xi,L_e)\, F_e(L_e,t^f)$$
$$- h_e(\xi,L_{e-1})\, F_e(L_{e-1},t^f) + u_e(\xi) + w_e(\xi) \tag{29}$$

where

$$h_e(\xi,x) = \frac{1}{c_e}\int_{t^{f-1}}^{t^f} J_e^* dt = \frac{\operatorname{sgn}(x-\xi)}{2} \operatorname{erfc}\left[\frac{|x-\xi|}{2\sqrt{a_e\Delta t}}\right] \tag{30}$$

$$g_e(\xi,x) = \frac{1}{c_e}\int_{t^{f-1}}^{t^f} F_e^* dt = \frac{\sqrt{\Delta t}}{\sqrt{\pi}\lambda_e c_e} \exp\left[-\frac{(x-\xi)^2}{4a_e\Delta t}\right]$$
$$- \frac{|x-\xi|}{2\lambda_e} erfc\left[\frac{|x-\xi|}{2\sqrt{a_e\Delta t}}\right] \tag{31}$$

$$u_e(\xi) = \int_{L_{e-1}}^{L_e} F_e^*(\xi,x,t^f,t^{f-1})\, F_e(x,t^{f-1})\, dx$$
$$= \frac{1}{2\sqrt{\pi a_e \Delta t}} \int_{L_{e-1}}^{L_e} \exp\left[-\frac{(x-\xi)^2}{4a_e\Delta t}\right] F_e(x,t^{f-1})\, dx \tag{32}$$

and

$$w_e(\xi) = \int_{L_{e-1}}^{L_e} S_e(x,t^{f-1})\, g_e(\xi,x)\, dx \tag{33}$$

For $\xi \to L_{e-1}^{+}$ and $\xi \to L_e^{-}$ for each sub-domain considered one obtains the system of equations

$$\begin{bmatrix} g_e(L_{e-1},L_{e-1}) & g_e(L_{e-1},L_e) \\ g_e(L_e,L_{e-1}) & g_e(L_e,L_e) \end{bmatrix} \begin{bmatrix} J_e(L_{e-1},t^f) \\ J_e(L_e,t^f) \end{bmatrix} = \begin{bmatrix} h_e(L_{e-1}^{+},L_{e-1})-1 & h_e(L_{e-1}^{+},L_e) \\ h_e(L_e^{-},L_{e-1}) & h_e(L_e^{-},L_e)-1 \end{bmatrix} \begin{bmatrix} F_e(L_{e-1},t^f) \\ F_e(L_e,t^f) \end{bmatrix} + \begin{bmatrix} u_e(L_{e-1}) \\ u_e(L_e) \end{bmatrix} + \begin{bmatrix} w_e(L_{e-1}) \\ w_e(L_e) \end{bmatrix} \tag{34}$$

or

$$\begin{bmatrix} g_{11}^e & g_{12}^e \\ g_{21}^e & g_{22}^e \end{bmatrix} \begin{bmatrix} J_e(L_{e-1},t^f) \\ J_e(L_e,t^f) \end{bmatrix} = \begin{bmatrix} h_{11}^e & h_{12}^e \\ h_{21}^e & h_{22}^e \end{bmatrix} \begin{bmatrix} F_e(L_{e-1},t^f) \\ F_e(L_e,t^f) \end{bmatrix} + \begin{bmatrix} u_e(L_{e-1}) \\ u_e(L_e) \end{bmatrix} + \begin{bmatrix} w_e(L_{e-1}) \\ w_e(L_e) \end{bmatrix} \tag{35}$$

The continuity conditions for $x = L_1$ and $x = L_2$ can be written in the form

$$x = L_1: \begin{cases} F_1(L_1,t^f) = F_2(L_1,t^f) = F_b(t^f) \\ J_1(L_1,t^f) = J_2(L_1,t^f) + A(L_1,t^f) \end{cases} \tag{36}$$

and

$$x = L_2: \begin{cases} F_2(L_2,t^f) = F_3(L_2,t^f) = F_d(t^f) \\ J_2(L_2,t^f) = J_3(L_2,t^f) + B(L_2,t^f) \end{cases} \tag{37}$$

In the case of basic problem $A(L_1,\ t^f) = 0$ and $B(L_2,\ t^f) = 0$, for the sensitivity problems these functions result from the boundary conditions for basal layer and dermal base (see: equations (7) and (16)). The final form of resolving system results from above continuity conditions and the conditions given for x =0 and $x = L$. So, for $t \le t_0$ we have

$$\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 & 0 & 0 \\ -h_{21}^1 & -h_{22}^1 & g_{22}^1 & 0 & 0 & 0 \\ 0 & -h_{11}^2 & g_{11}^2 & -h_{12}^2 & g_{12}^2 & 0 \\ 0 & -h_{21}^2 & g_{21}^2 & -h_{22}^2 & g_{22}^2 & 0 \\ 0 & 0 & 0 & -h_{11}^3 & g_{11}^3 & -h_{12}^3 \\ 0 & 0 & 0 & -h_{21}^3 & g_{21}^3 & -h_{22}^3 \end{bmatrix} \begin{bmatrix} F_1(0,t^f) \\ F_b(t^f) \\ J_1(L_1,t^f) \\ F_d(t^f) \\ J_2(L_2,t^f) \\ F_3(L,t^f) \end{bmatrix} = \begin{bmatrix} C(0,t^f) + u_1(0) + w_1(0) \\ C(0,t^f) + u_1(L_1) + w_1(L_1) \\ g_{11}^2 A(L_1,t^f) + u_2(L_1) + w_2(L_1) \\ g_{21}^2 A(L_1,t^f) + u_2(L_2) + w_2(L_2) \\ g_{11}^3 B(L_2,t^f) + u_3(L_2) + w_3(L_2) \\ g_{21}^3 B(L_2,t^f) + u_3(L) + w_3(L) \end{bmatrix} \tag{38}$$

The value of $C(0,t^f)$ in the last equation is connected with the type of boundary condition given for x=0 (as in formula (6)).

The internal values of functions F_e corresponding to time t^f are calculated for the each layer of tissue, separately using the formula

$$F_e(\xi,\ t^f) = g_e(\xi,\ L_{e-1}) J_e(L_{e-1},t^f) - g_e(\xi,\ L_e) J_e(L_e,t^f) + h_e(\xi,\ L_e) F_e(L_e,t^f) - h_e(\xi,\ L_{e-1}) F_e(L_{e-1},t^f) + u_e(\xi) + w_e(\xi) \tag{39}$$

5 Results of computations

In numerical computations the following mean values of parameters have been assumed: $\lambda_1 = 0.235$ [W/mK], $\lambda_2 = 0.445$ [W/mK], $\lambda_3 = 0.185$ [W/mK], $c_1 = 4.3068 \cdot 10^6$ [J/m^{3}K], $c_2 = 3.96 \cdot 10^6$ [J/m^{3}K], $c_3 = 2.674 \cdot 10^6$ [J/m^{3}K], $c_B = 3.9962 \cdot 10^6$ [J/m^{3}K], $G_{Be} = 0.00125$ [m^3 blood/s/ m^{3}tissue], $e = 2,\ 3$, $G_{B1} = 0$, $T_B = 37°$C, $Q_{me} = 245$ [W/m^3] for $e = 2,\ 3$, while $Q_{m1} = 0$ (Torvi and Dale (1994)). The thicknesses of successive skin layers: 0.1, 2 and 10 [mm]. Pre-exponential factors $P_b = 1.43 \cdot 10^{72}$, $P_d = 2.86\text{A}10^{69}$ [1/s] for $T_b \ge 317$K and $Pb = P_d = 0$ for $T_b < 317$K. The initial temperature distribution has been assumed to be the parabolic one (as in Figure 2). The skin subdomains have been divided into 10, 40 i 120 linear internal cells of dimensions 10^{-5}, $5{\cdot}10^{-5}$, $8.333{\cdot}10^{-5}$ [m]), respectively. The computations have been realized with time step Δt =0.05 [s].

The first example concerns the thermal processes in the tissue subjected to the boundary heat flux q_0 =6500 [W/m^2] and the exposure time t_0 =18 [s]. For t¿t_0 the Robin condition is assumed (α =8 [W/m^{2}K], T^∞ =20^{o}C).

The temperature field obtained for above input data is shown in Figure 2. On the basis of the knowledge of $T(L_1,t)$ (basal layer) and $T(L_2,t)$ (dermal base) we found that the times to first and second degree burn are equal 16.1 [s] and 17.55 [s], respectively. The third degree burn in this case does not appear.

Next, the sensitivity analysis of temperature field and burn integral has been done with respect to the all thermophysical parameters.

The part of the results obtained is shown in Figures 3, 4, 5, 6. In Figure 3 and 4 the distribution of sensitivity functions $\partial T/\lambda_1$ and $\partial T/c_2$ are marked. The changes of the epidermis thermal conductivity are not very essential for the course of the burn integral I_b. Figure 5 presents the course of I_b for the values of λ_1 from the interval [0.21, 0.26 W/mK]. One can see that the times of the burns apparitions are practically the same. The other situation takes place in the case of the parameter c_2. For $\Delta c_2 = 120000$ [J/m^{3}K] we observe the essential changes of integral I_b- Figure 6. So, the influence of parameters on the course of thermal processes in the tissue domain is distinctly discriminated (in numerical realization we assumed $\Delta p_s/p_s$=const).

The results above presented have been obtained using the first-order sensitivity analysis.

The second example concerns the thermal processes in the tissue subjected to the boundary heat flux $q_0 = 90000$ [W/m^2] and the exposure time $t_0 = 4$ [s]. The remaining input data are the same. Because of the big gradient of temperature and heating rate, the second-order sensitivity has been considered. The computations previously discussed showed that the influence of the dermis conductivity (λ_2) on the heat transfer process is also very essential and the results of the second-order analysis concerning this parameter will be below presented.

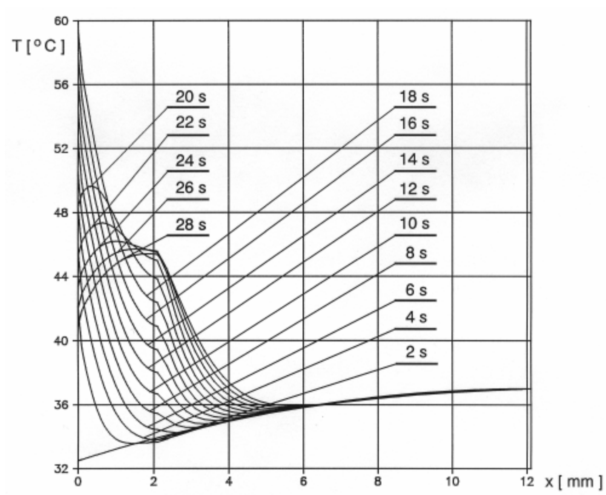

Figure 2 : Temperature distribution

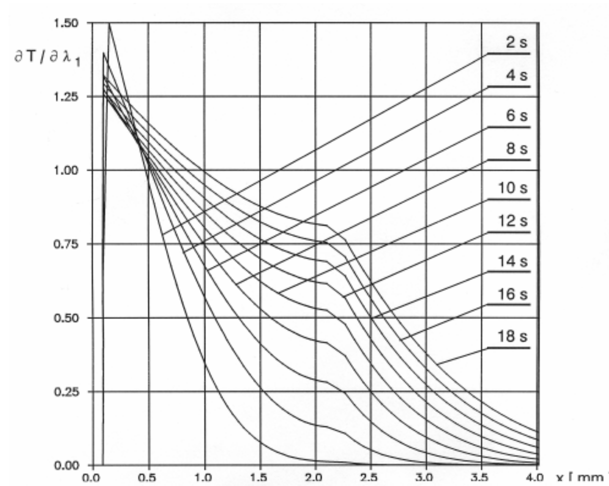

Figure 3 : Distribution of function $\partial T/\partial\lambda_1$ for $\chi \subset$ [0,4 mm]

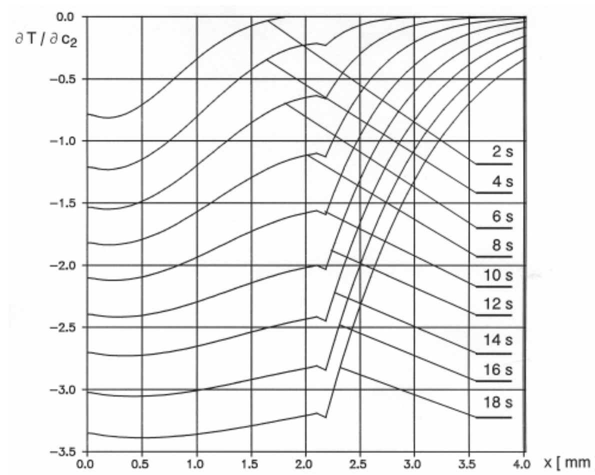

Figure 4 : Distribution of function $\partial T/\partial c_2$

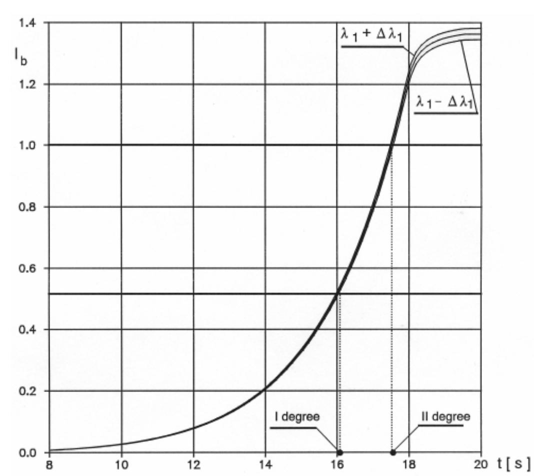

Figure 5 : Course of burn integral and its sensitivity with respect to λ_1

In Figure 7 the basic solution, this means the temperature field in the skin tissue is shown. The next Figure illustrates the distribution of the derivative $\partial^2 T/\partial\lambda_2^2$. The courses of the burn integral I_d for the values of λ_2 from the scope [0.42, 0.47 W/mK] are marked in Figure 9. One can see that the time of the third degree burn apparition is considerable differentiated. In order to verify the exactness of the algorithm proposed the extreme values of λ_2 have been directly introduced to the procedure solving the basic problem and the differences between solutions were very small.

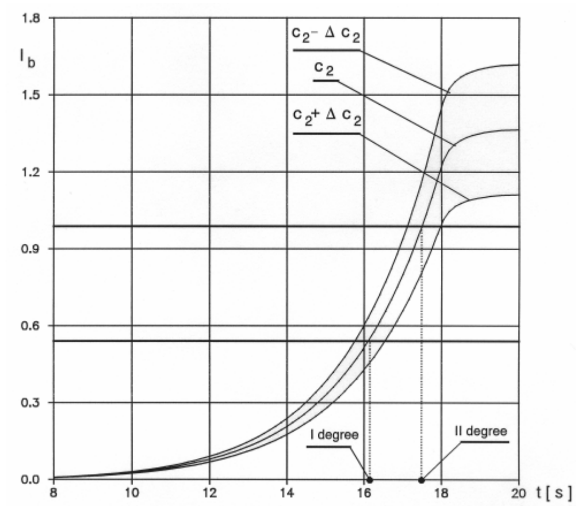

Figure 6 : Course of burn integral I_b and its sensitivity with respect to c_2

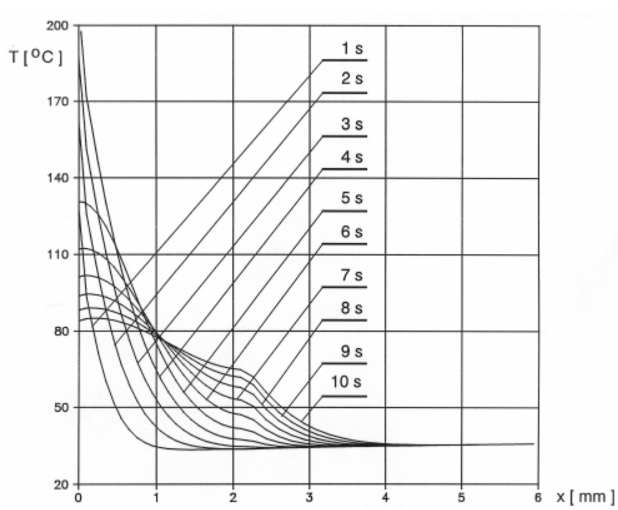

Figure 7 : Temperature field

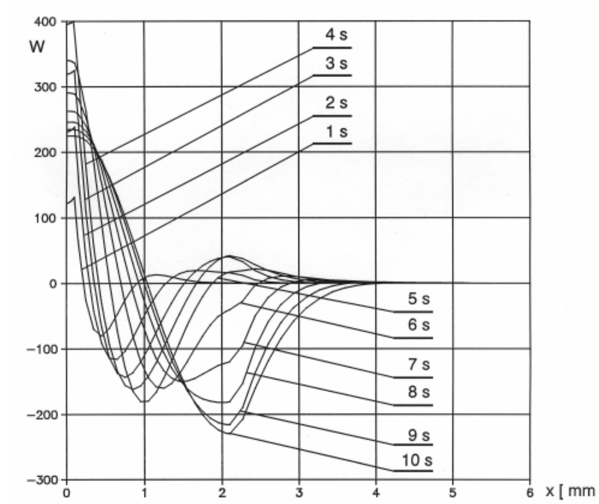

Figure 8 : Distribution of function $\partial^2 T/\partial\lambda_2^2$

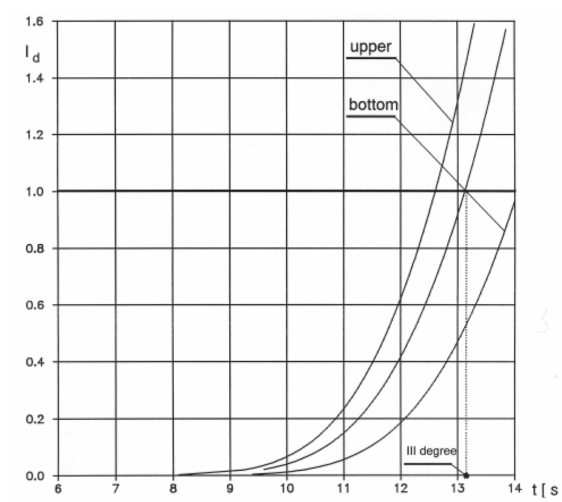

Figure 9 : The change of I_d with respect to λ_2

6 Conclusions

It seems that the application of the boundary element method to the solution of the problem discussed leads to the very effective numerical algorithm. The number of unknown parameters is not large (only the boundary values). Next, the internal temperatures and the internal values of functions U, W can be found at the optional set of points on the basic of the knowledge of boundary ones. The approximation of the real boundary conditions is exact and it causes that the solution in the interior of domain is sufficiently exact, too.

The results of computations presented in this paper show that the influence of different thermophysical parameters on the course of the process is discriminated. The quantitative valuation of these problems can be done first of all on the basis of sensitivity analysis. The quite good results can be, as a rule, obtained using the first-order approach, while the second-order analysis is useful in the case of parameters especially responsible to the course of thermal processes.

The next stage of the investigations in this scope will concern the sensitivity analysis with respect to geometrical parameters. Especially the epidermis thickness can have the essential importance on the thermal damage of the skin.

Acknowledgement: The paper was sponsored by KBN (Grant No 8 T11F 004 19).

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