Advances in Finite Rotations in Structural Mechanics

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In 1995, Atluri and Cazzani(1995) published a review paper entitled: "Rotations in Computational Solid Mechanics." The mathematical aspects of finite rotations have been explained, and the role of finite rotations in both continuum mechanics and multi-rigid body dynamics has been presented in that paper. Several variational principles developed by Atluri and his colleagues, staring 1984[Atluri(1984)] involve finite rotations, and its tensor and vector representations, directly as variables. The finite element formulations based on these variational principles always lead to the symmetric tangent stiffness matrix. The numerical examples shown in various subsequent literature, have revealed the efficiency of their formulation. The extensive list of references can be found in the paper, Atluri and Cazzani(1995).

Before and even after the work of Atluri and Cazzani, many attempts have been made to treat finite rotations. Finite rotations have a long history in rigid-body dynamics. A variety of methods for treating finite rotations can be found in text books [see, for example, Kane, Likins and Levinson(1983) or Wittenburg (1977)]. Several useful systems for representing finite rotations are Euler angles, Bryant angles, Euler parameters and Rodrigues parameters. The choice of the system depends on the problem to be solved. A variety of reference frames has been proposed for formulating the rigid-body dynamics. One of the key points for determining the reference frames is how simply constraint conditions are satisfied. These topics have been discussed along with efficient solvers of differential algebraic equations [see, for example, Haug and Deyo(2001)].

Since the mid-seventies, special attention has been attracted to flexible bodies such as beams, plates and shells, involving finite rotations. The total Lagrangian formulation or up-dated Lagrangian formulation have often been used in continuum mechanics. The early developments in treating finite rotations as direct independent variables in the variational formulations of finitely deformed (finite streches and finite rotations) continua and shelles can be found in the paper of Atluri (1984) and

also in Pietraszkiewicz (1986). The need to solve complicated structures requires more sophisticated strategies. It is natural, therefore, to combine the finite element technology and rigid-body dynamic technology for efficient analysis of flexible multi-body systems. Recent developments in flexible multi-body systems are contained in the symposia proceedings [see, for example, Banichuk, Klimov and Schiehlen (1990), and Ambrosio Kleiber(2001)]. Along with the development of kinematic formulations, numerical integration schemes of dynamic equations have been proposed to obtain more accurate and stable solutions [see, for example, Rochinha and Sampaio (2000)]. Integrated systems may be required for efficient analysis of flexible multi-body dynamics.

The present special issue deals with finite rotations in beams, plates and shells. Lin and Hsiao(2003) solved buckling problems of 3-D beams by using the co-rotational formulation. The element coordinates were used by Ijima, Obiya, Iguchi and Goto(2003) to solve large displacement problems of space frames. Goto, Kuwataka, Nishihara and Iwakuma(2003) introduced rotational angles associated with the Cartesian coordinates. Although these angles can not express finite rotations, their infinitesimal rotations are related with Euler angles. In the papers mentioned above, the so-called co-rotational formulation has been used. The accuracy of the co-rotational formulation is discussed from a theoretical point of view by Iura, Suetake and Atluri(2003). They have introduced a new coordinate system which is efficient in the co-rotational formulation.

The total Lagrangian formulation is introduced by Zupan and Saje(2003), and Beda.(2003), and Zupan and Saje(2003) use Rodrigues parameters to express finite rotations, in which the pseudo-curvature vector played an important role. Beda(2003) introduces three rotational angles associated with coordinates fixed in a space, and solves the elastica problem of spatial Euler-Bernoulli beam. Okamoto and Omura(2003) use the up-dated Lagrangian formulation for dynamic analysis of flexible

structures. The application of a cantilever beam moving along a nonlinear trajectory is demonstrated.

Symmetric tangent stiffness operators for thick shells undergoing finite rotations Are derived by Suetake, Iura and Atluri(2003). The symmetry of the tangent operator is discussed in the context of the conventional Gateaux derivative. Briseghelle, Majorana and Pavan(2003) develop the energy and momentum conserving algorithm to integrate the dynamic equations of elasto-damaged thin shells. Basar and Kintzel(2003) develop an updated-rotational formulation for smooth and compound shells, in which they derive symmetric tangent stiffness operators.

Shape optimization of elastic structural systems was analyzed by Ibrahimbegovic and Knopf-Lenoir(2003) . A simultaneous solution procedure for analysis ,and design leading to optimal shape was presented.

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