

## Modeling of Nonlinear Rate Sensitivity by Using an Overstress Model

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**Abstract:** Negative, zero or positive rate sensitivity of the flow stress can be observed in metals and alloys over a certain range of strain, strain rate and temperature. It is believed that negative rate sensitivity is an essential feature of dynamic strain aging, of which the Portevin-Le Chatelier effect is one other manifestation. The viscoplasticity theory based on overstress (VBO), one of the unified state variable theories, is generalized to model zero (rate independence) and negative as well as positive rate sensitivity in a consistent way. The present model does not have the stress rate term in the evolution law for the state variable equilibrium stress that has been included in previous versions of VBO. The three types of rate sensitivity are classified on the basis of a certain constant of an augmentation function, which is introduced in the evolution law for the equilibrium stress. Based on the augmentation function, the model reproduces the dependence of relaxation rate on prior strain rate. When the augmentation function is selected to depend on the accumulated inelastic strain or the effective inelastic strain rate, a change of rate sensitivity type that depends on strain or strain rate can be modeled. It is also shown that the augmentation function can be used to reproduce the observed dramatic increase of strain hardening at strain rates exceeding  $10^3 \text{ s}^{-1}$ .

**keyword:** Constitutive Behavior; Viscoplasticity; Dynamic Strain Aging; Stress Relaxation; Dynamic Plasticity.

### 1 Introduction

Numerous investigations of constitutive models to represent rate dependent inelastic deformation behavior of materials have been performed in the past decades. Although constitutive modeling has made considerable progress, it seems to be confined to some specific

deformation behaviors [see, for instance, Tvergaard and Pedersen(2000), and Kailasam, Aravas, and Ponte Casañeda(2000)]. Thus the need for improved constitutive models to represent the variety of deformation behaviors that can be found in many metals and alloys has led to this work that is a generalization and simplification of the viscoplasticity theory based on overstress.

Metals and alloys exhibit dynamic strain aging behavior that is manifested by serrated flow, zero or negative rate sensitivity, pronounced strengthening and reduction of ductility in a certain range of temperature, see Mulford and Kocks (1979) and Kishore et al. (1997), to name just a few. A significant amount of experimental and theoretical study has been performed to establish microscopic mechanisms responsible for dynamic strain aging since Cottrell (1953) proposed a model based on interaction of diffusing solute atoms with mobile dislocations. It is generally recognized that metals and alloys experience dynamic strain aging in some temperature region. When this happens negative rate sensitivity is a prerequisite for serrated flow that depends on strain and/or strain rate, see Penning (1972), Miller and Sherby (1978), Mulford and Kocks (1979) and Kalk and Schwink (1992).

In the framework of the viscoplasticity theory based on overstress, developed by Krempl and his colleagues, the constitutive model is generalized and is simplified so that positive, zero and negative rate sensitivity of the flow stress including other inelastic deformation behaviors can be modeled easily. The purpose of this work is not a study of the microscopic mechanism related to negative rate sensitivity but the phenomenological modeling of the three different types of rate sensitivity together with rate dependent deformation behavior. The modeling capability is easily extended to reproduce a change of strain rate sensitivity depending on strain or strain rate.

Experiments have shown the nonlinear dependence of the relaxation rate on prior strain rate. The test associated with the fastest prior strain rate has the smallest stress magnitude at the end of the same relaxation period,

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see Bordonaro and Krempl (1992), Majors and Krempl (1994), Ariyama et al. (1997), Yaguchi and Takahashi (1997) and Krempl and Nakamura (1998).

In dynamic plasticity some materials can exhibit a dramatic increase of rate sensitivity as the strain rate exceeds  $10^3 \text{ s}^{-1}$ , see Stout and Follansbee (1986), Follansbee and Kocks (1988), Clifton (1990) and Bodner and Rubin (1994). Motivated by the fact that the strain rate dependence is a crucial feature at very high strain rates, it is of interest to include the effect in constitutive modeling.

In the following the generalized viscoplasticity theory is first investigated for a hypothetical material to demonstrate the modeling capabilities and then applied to reproduce the observed inelastic deformation behaviors of 304L stainless steel tested by Stout and Follansbee (1986) and a modified 9Cr-1Mo steel investigated by Yaguchi and Takahashi (1999).

## 2 The Proposed Model

A generalized viscoplasticity theory based on overstress, one of the unified state variable theories, does not have separate repositories for plastic and creep deformation, a yield criterion and a loading/unloading condition. The constitutive equations consist of a set of nonlinear, coupled ordinary differential equations; one flow law and one evolution law for each of three state variables that represent some features of the microstructural state and its change.

It is known that inelastic deformation of materials is considered a rate dependent process and during deformation materials have two competing mechanisms, hardening and dynamic recovery. The second law of thermodynamics, together with the two competing mechanisms, have provided a good framework in constructing constitutive equations, see Swearngen et al. (1985), Freed et al. (1991), Chaboche (1993) and Krempl (1996). In accordance with the notion, the proposed model eliminates the stress rate term in the evolution law for the state variable equilibrium stress that has been included in the previous formulation by Ho (1998) and Ho and Krempl (2001); the stress rate term has been found to be responsible for difficulties in showing thermodynamic consistency, see Krempl (1996).

Materials exhibit different endpoints of relaxation tests of equal duration at the same strain but with different prior

strain rate. When the equilibrium (back) stress is rate independent in fully established inelastic flow region on loading, this relaxation property cannot be modeled without the stress rate term in the evolution law; the equilibrium stress changes and thus becomes rate dependent during relaxation period due to the stress rate term, see Krempl and Nakamura (1998). However, the proposed model can surely reproduce the relaxation property without the stress rate term through making the equilibrium stress rate dependent in fully established inelastic flow region.

The total strain rate is assumed to be the sum of the elastic and the inelastic strain rate. The elastic part is formulated from the rate form of the generalized Hooke's law and the inelastic part is a function of the overstress, the difference between the stress and the tensor-valued state variable equilibrium stress. The overstress plays a key role as a repository of nonlinear rate dependency and the equilibrium stress can be considered as the history dependent stress sustained at rest. The model has two additional state variables; the tensor-valued kinematic stress and the scalar isotropic stress. The purpose of the kinematic stress is to model the slope of the stress-strain diagram for fully established inelastic flow. The isotropic stress is responsible for the modeling of cyclic hardening or softening.

### 2.1 Multiaxial formulation

We assume small strain, isotropic and incompressible inelastic deformation under isothermal condition. For a multiaxial formulation, the flow law is taken to be

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{el} + \dot{\varepsilon}_{ij}^{in} = \left( \frac{1+\nu}{E} \dot{\sigma}_{ij} - \frac{\nu}{E} \delta_{ij} \dot{\sigma}_{kk} \right) + \left( \frac{3}{2Ek} X_{ij} - \frac{1}{2Ek} \delta_{ij} X_{kk} \right) \quad (1)$$

where  $\sigma_{ij}$  and  $\varepsilon_{ij}$  are the stress and strain tensors, respectively;  $E$  and  $\nu$  are the elastic modulus and Poisson's ratio, respectively. A superposed dot denotes material time derivative,  $\delta_{ij}$  is the Kronecker delta and  $X_{ij}$  is the overstress defined by  $\sigma_{ij} - g_{ij}$ . The inelastic strain rate is taken to be an increasing function of the overstress that is the difference between the stress and the equilibrium stress. The viscosity function  $k$  is introduced to model

nonlinear rate dependence and is a positive, decreasing, continuous function of the overstress invariant  $\Gamma$  given by

$$\Gamma = \left( \frac{3}{2} X_{ij} X_{ij} - \frac{1}{2} X_{ii} X_{jj} \right)^{1/2} = \left( \frac{3}{2} X_{ij}^d X_{ij}^d \right)^{1/2} \quad (2)$$

where  $X_{ij}^d = \sigma_{ij}^d - g_{ij}^d$  is the deviatoric overstress defined by the deviator of the stress and the equilibrium stress.

The evolution law for the equilibrium stress is

$$\dot{g}_{ij} = \psi \left[ \frac{X_{ij}}{Ek} - \frac{(g_{ij} - f_{ij})}{A_c + |A + \beta\Gamma|} \dot{\phi} \right] + \dot{f}_{ij} - r g_{ij} \quad (3)$$

where  $r$  and  $A_c$  are positive material constants. The effective inelastic strain rate is given by

$$\dot{\phi} = \left( \frac{2}{3} \dot{\epsilon}_{ij}^{in} \dot{\epsilon}_{ij}^{in} \right)^{1/2} = \frac{\Gamma}{Ek} \quad (4)$$

The augmentation function  $\beta$  is newly introduced into the dynamic recovery term of the evolution law for the equilibrium stress so as to make the equilibrium stress rate dependent. In such a way negative, zero and positive rate sensitivity of the flow stress can be easily modeled. The material constant  $\psi > 0$  influences the transition between initial quasi-elastic behavior and fully established inelastic flow. The static recovery term  $r g_{ij}$  represents the softening caused by thermal diffusion at elevated temperature, see Choi (1989) and Tachibana and Krempl (1995).

In contrast to the previous version by Ho (1998) and Ho and Krempl (2001), the evolution law for the equilibrium stress has no stress rate dependence and the denominator of the dynamic recovery term,  $A_c + |A + \beta\Gamma|$ , is positive all the time to confirm the recovery role competing with the hardening contribution. From Eq. (1), the stress becomes equal to the equilibrium stress when  $\dot{\sigma}_{ij} = \dot{\epsilon}_{ij} = 0$ . The equilibrium stress can therefore be identified as the stress sustained at rest. When the equilibrium stress is set to have negative rate sensitivity through a properly defined augmentation function and rarely changes during relaxation period the present version may reproduce realistic relaxation behavior, which means that the relaxed stress associated with the fastest prior strain rate has the smallest magnitude at the end of the same relaxation period. This modeling capability will be demonstrated in the next section.

Through the competing effect of the strain hardening and the dynamic recovery, the theory also allows to model the change of hardening rate and strain rate sensitivity with straining, and the cyclic hardening followed by softening or vice versa, see Ho (1998) for details.

The evolution law for the kinematic stress is

$$\dot{f}_{ij} = E_t \frac{X_{ij}}{Ek} \quad (5)$$

where  $E_t$  is related to the tangent modulus based on total strain  $\hat{E}_t$  by  $\hat{E}_t = E_t / (1 + E_t/E)$ .

The positive, scalar isotropic stress has the following evolution law

$$\dot{A} = A_r (A_f - A) p; \quad p = \int \dot{\phi} dt \quad (6)$$

with an initial value,  $A(t=0) = A_0$ . In the above  $p$  is the accumulated inelastic strain,  $A_f$  is the final value of  $A$  and  $A_r$  is the material constant that influences the evolution rate of the isotropic stress.

The viscosity function is taken to be

$$k = k_1 \left( 1 + \frac{\Gamma}{k_2} \right)^{-k_3} \quad (7)$$

where  $k_1$ ,  $k_2$  and  $k_3$  are material constants.

## 2.2 Reduction to the uniaxial state of stress

To motivate the interpretation of the state variables of the theory, the uniaxial formulation is shown below. Using  $\Gamma = |\sigma - g|$  and  $\dot{\phi} = |\dot{\epsilon}^{in}|$ , we have

$$\dot{\epsilon} = \dot{\epsilon}^{el} + \dot{\epsilon}^{in} = \frac{\dot{\sigma}}{E} + \frac{(\sigma - g)}{Ek[\Gamma]} \quad (8a)$$

$$\dot{g} = \psi \left[ \frac{(\sigma - g)}{Ek} - \frac{(g - f)}{A_c + |A + \beta\Gamma|} \left| \frac{\sigma - g}{Ek} \right| \right] + \dot{f} - r g \quad (8b)$$

$$\dot{f} = E_t \frac{(\sigma - g)}{Ek} \quad (8c)$$

For a constant strain rate test, the constitutive equations admit asymptotic solutions as the time grows without bounds, see Appendix for details. The asymptotic solutions can be obtained by the integral representation converted from the differential form of the constitutive equations, see Cernocky and Krempl (1979) and Ho (1998). These solutions are recognized to correspond to the fully established inelastic deformation and give rise to essential features of the theory.

For the asymptotic state, we obtain

$$\left\{ \frac{d\sigma}{d\varepsilon} \right\} = \left\{ \frac{dg}{d\varepsilon} \right\} = \left\{ \frac{df}{d\varepsilon} \right\} \quad (9)$$

and

$$\{\dot{A}\} = 0 \quad (10)$$

where brace denotes the asymptotic value of the quantity. Based on these basic relations, we can rewrite Eq. (8a) as

$$\{\sigma - g\} = \frac{E}{1 + E_t/E} \{k[\Gamma]\} \dot{\varepsilon} \quad (11)$$

The overstress in the asymptotic state depends nonlinearly on the strain rate due to the presence of the viscosity function and always increases nonlinearly with an increase of the strain rate.

An asymptotic solution for the difference  $g - f$  is

$$\{g - f\} = \{A_c + |A_f + \beta\Gamma|\} \left\{ \frac{\sigma - g}{\Gamma} \right\} \quad (12)$$

and we then obtain

$$\{\sigma - f\} = \{A_c + |A_f + \beta\Gamma| + \Gamma\} \left\{ \frac{\sigma - g}{\Gamma} \right\} \quad (13)$$

The stress minus the kinematic stress consists of the rate independent (plastic) and the rate dependent (viscous) contributions given by

$$\{\sigma - f\} = \begin{cases} \{A_c + A_f\} \{(\sigma - g)/\Gamma\} \\ \{A_c - A_f\} \{(\sigma - g)/\Gamma\} \end{cases} \quad (14)$$

$$+ \begin{cases} \{(1 + \beta)\Gamma\} \{(\sigma - g)/\Gamma\} & \text{for } A_f + \beta\Gamma \geq 0 \\ \{(1 - \beta)\Gamma\} \{(\sigma - g)/\Gamma\} & \text{for } A_f + \beta\Gamma < 0 \end{cases}$$

It should be noted that the kinematic stress is rate independent and the rate independent isotropic stress  $A_f$  and the overstress invariant  $\Gamma$  are defined to be positive. When  $A_f + \beta\Gamma \geq 0$ , the three types of rate sensitivity associated with stress-strain diagram can be distinguished as follows:

$$\beta < -1 \quad ; \text{ Negative rate sensitivity,} \quad (15a)$$

$$\beta = -1 \quad ; \text{ Zero rate sensitivity} \quad (15b)$$

$$\beta > -1 \quad ; \text{ Positive rate sensitivity.} \quad (15c)$$

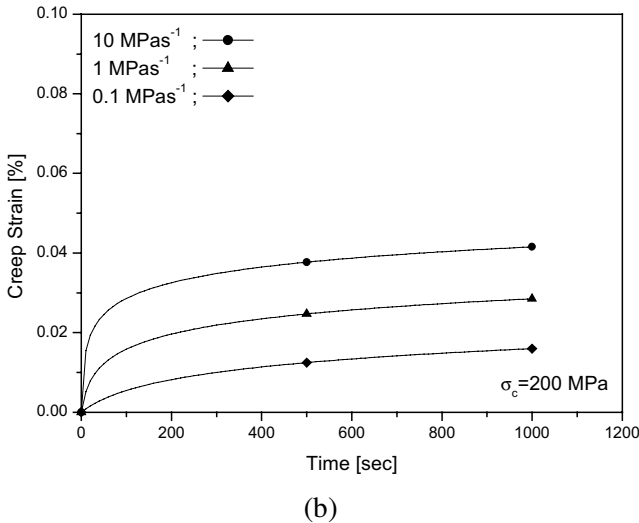
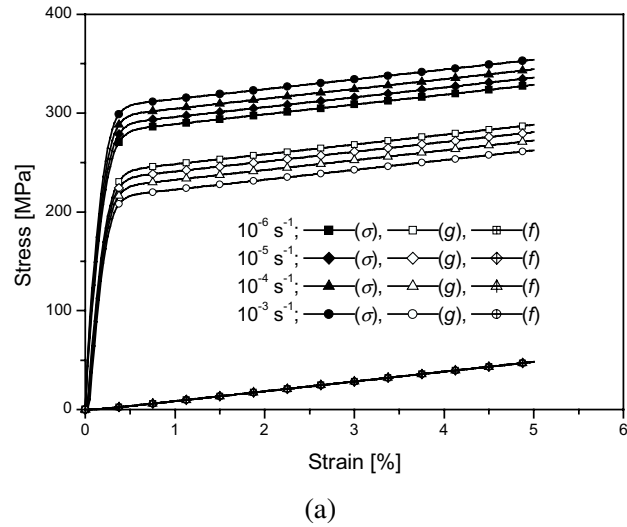
Setting  $\beta = -1$  eliminates the rate dependent contribution so that the flow stress level shows zero rate sensitivity (rate independence). The conditions  $\beta < -1$  and  $\beta > -1$  correspond to negative and positive contributions of the rate dependent term to the flow stress level, respectively, since the overstress always increases with strain rate. On the other hand, when strain rate becomes large enough to satisfy  $A_f + \beta\Gamma < 0$  the rate dependent term always has the positive contribution to the flow stress and only positive rate sensitivity of the flow stress is thus reproduced. The constitutive equations can therefore model negative or zero rate sensitivity depending on strain rate in a natural way with no constraint.

### 3 Numerical simulations

First the generalized viscoplasticity theory is demonstrated for a hypothetical material to depict the qualitative modeling capabilities. The theory is then applied to experiments on a modified 9Cr-1Mo steel tested by Yaguchi and Takahashi (1999) and a 304L stainless steel tested by Stout and Follanbee (1986). In performing most cases, the isotropic stress is kept constant ( $A_r = 0$ ) in a uniaxial test since the isotropic stress is responsible for modeling of cyclic hardening or softening behaviors; in fact, the accumulated inelastic strain is so small that the evolution of the isotropic stress may be neglected in a uniaxial test.

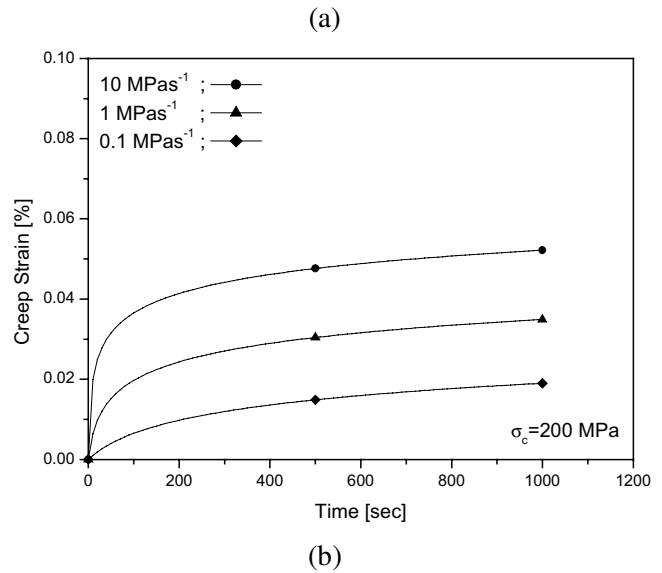
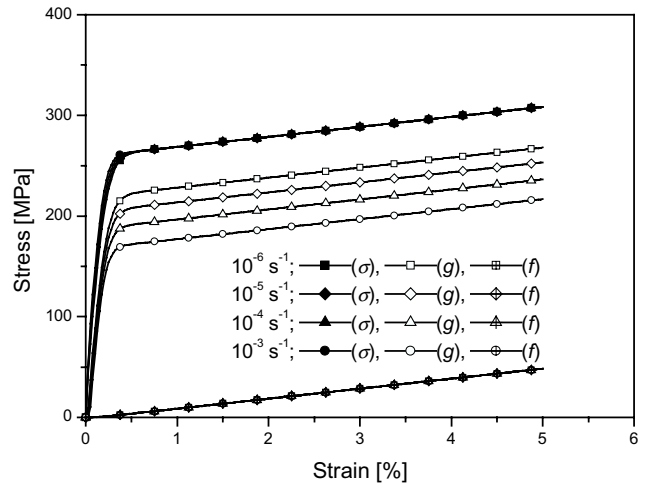
### 3.1 Hypothetical material

With the help of Eqs. (15a), (15b) and (15c), numerical simulations in uniaxial tensile loading are presented for a hypothetical material using the arbitrary material constants given in Table 1.



**Figure 1 :** Positive rate sensitivity of flow stress with  $\beta = -0.5$ . (a) Shown are the stress, equilibrium stress and kinematic stress; (b) Each creep test starts at 200Mpa.

Figure 1a shows positive rate sensitivity for  $\beta = -0.5$ , which means the flow stress levels increase nonlinearly with the strain rates. The stress, the equilibrium stress and the kinematic stress grow at the same rate in the fully developed inelastic flow region. The kinematic stress exhibits rate insensitivity and the overstress  $\sigma - g$  increases



**Figure 2 :** Zero rate sensitivity of flow stress with  $\beta = -1$ . (a) Shown are the stress, equilibrium stress and kinematic stress; (b) Each creep test starts at 200Mpa.

nonlinearly with the strain rate as expected from Eq. (11). The equilibrium stress is not unique and decreases nonlinearly with the strain rate.

Figures 2a and 3a depict zero rate sensitivity with  $\beta = -1$  and negative rate sensitivity with  $\beta = -1.5$ , respectively. In both cases, as well as for positive rate sensitivity, the overstress increases with an increase of the strain rate since the asymptotic value of  $(\sigma - g)$  is independent of  $\beta$ , see Eq. (11). This property causes that regardless of the type of strain rate sensitivity of the flow stress, the stress drop at the end of relaxation tests with constant duration

increases nonlinearly with prior strain rate.

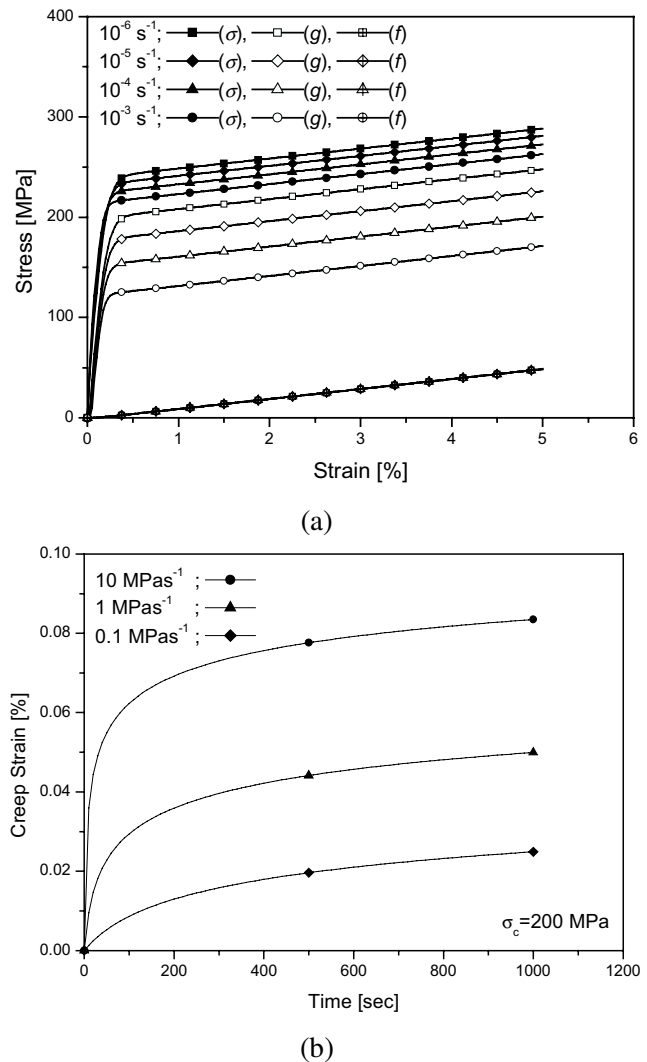
As the manifestation of rate sensitivity, creep and relaxation behavior exhibit an increasing strain during a fixed stress and a decreasing stress during a fixed strain, respectively. One may consider creep to be counterpart of relaxation as the rate-dependent inelastic deformation behavior and thus materials exhibit an increase of creep strain with increasing prior stress rate, see Krempl (1979), Stouffer and Dame (1996), Ho (1998) and Irizarry-Quinones (1999). The dependence of the creep rate on prior stress rate is shown in Figs. 1b, 2b and 3b.

During relaxation test the stress approaches the equilibrium stress and when the stress catches up with the equilibrium stress the relaxation finally ends up as expected from Eq. (8a). Under the positive rate sensitivity of flow stress ( $\beta > -1$ ), Figs. 4, 5 and 6 show how the value of the augmentation function is related to the trend of relaxation behavior. The equilibrium stress exhibits zero, positive and negative rate sensitivity for  $\beta = 0$ ,  $\beta = 0.5$  and  $\beta = -0.5$ , respectively, as expected from Eqs. (11) and (12). The stress and the equilibrium stress versus the total strain are simulated in Figs. 4a, 5a and 6a.

When  $\beta = 0$  the equilibrium stress is rate independent in the fully developed inelastic flow region as shown in Fig. 4a. Figure 4b shows the relaxation behavior started at  $\epsilon = 0.03$  for the different prior strain rates. Since the equilibrium stress exhibited rate independence before the relaxation tests and rarely changes during the relaxation tests, the relaxed stresses for the different prior strain rates finally ends up at one point as shown in Fig. 4b. Figure 5a shows the positive rate sensitivity of the equilibrium stress and thus the relaxed stress of the fastest prior strain rate has the largest stress magnitude at the end of the relaxation periods as shown in Fig. 5b. From these results, we know that the case  $\beta \geq 0$  cannot reproduce realistic relaxation behavior.

On the other hand, for  $\beta = -0.5$  in Fig. 6b the relaxed stress associated with the fastest prior strain rate has the smallest magnitude due to the negative rate sensitivity of the equilibrium stress before the relaxation tests. It is thus true that only the case  $\beta < 0$  can model realistic relaxation behavior. In other words, if the augmentation function is estimated from relaxation test it should have a negative value.

So far the augmentation function  $\beta$  has been held con-

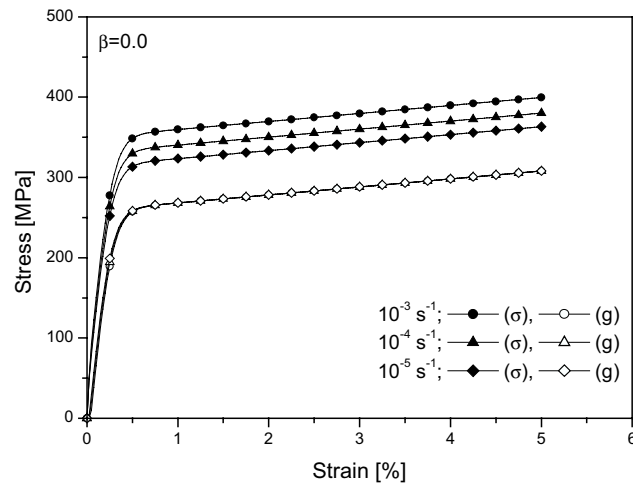


**Figure 3** : Negative rate sensitivity of flow stress with  $\beta = -1.5$ . (a) Shown are the stress, equilibrium stress and kinematic stress; (b) Each creep test starts at 200Mpa.

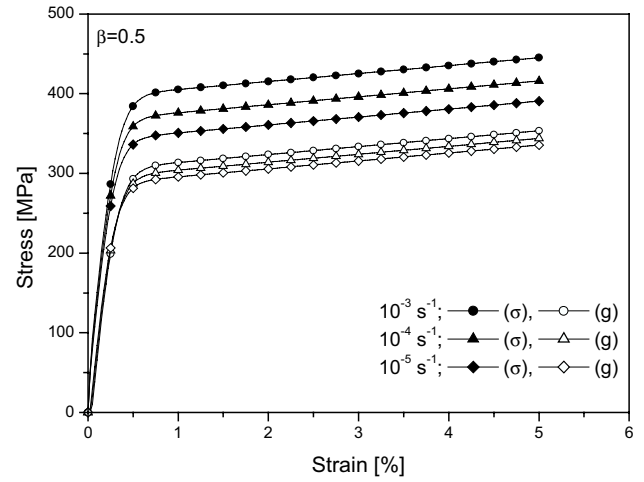
stant during deformation, but it can be chosen to change with straining so as to model the strain dependence of dynamic strain aging behavior, i.e., region of zero or negative rate sensitivity. To this end the augmentation function is taken to be

$$\beta = \beta_1 \exp(-\beta_2 p) + \beta_3 \tag{16}$$

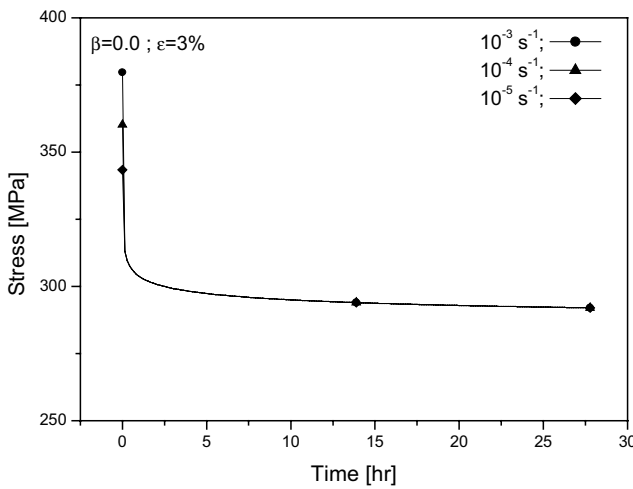
where  $p$  is the accumulated inelastic strain. Figures 7 and 8 show the transition of strain rate sensitivity with straining where the strain rate alternates between  $10^{-3}$



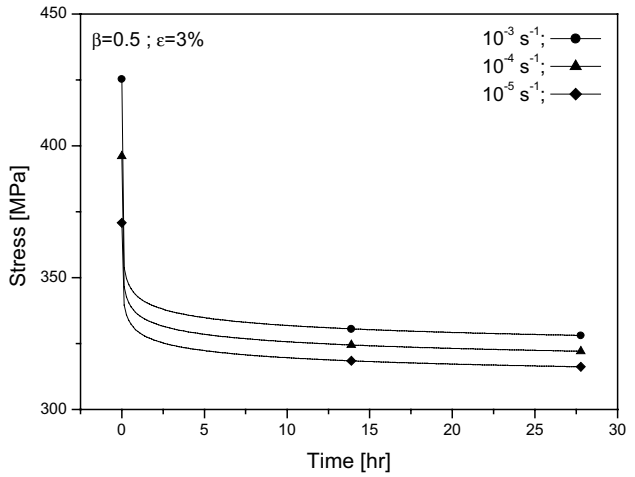
(a)



(a)



(b)



(b)

**Figure 4** : Relaxation behavior for  $\beta = 0$ . (a) Stress and equilibrium stress versus strain; (b) Stress at 3% strain versus time.

and  $10^{-6} \text{ s}^{-1}$ . When the augmentation function continuously decreases from  $\beta = -0.5$  to  $\beta = -1.5$ , the rate sensitivity changes from positive to negative as shown in Fig. 7. Figure 8 shows the negative rate sensitivity followed by the positive one with the increasing augmentation function.

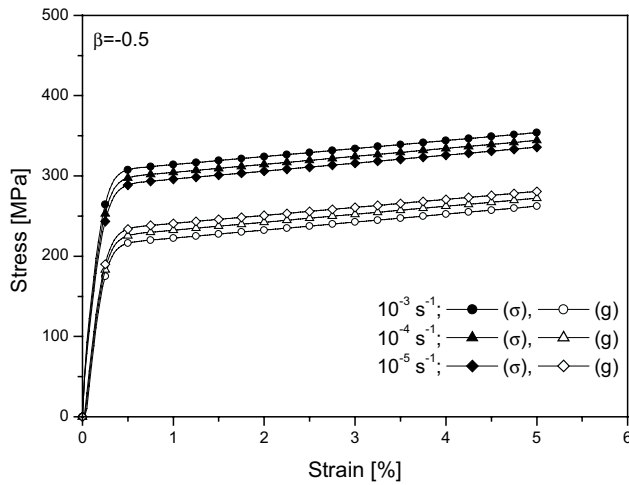
To see the effect of strain rate on negative rate sensitivity, the augmentation function is assumed to be the function of the effective inelastic strain rate as follows:

$$\beta = \beta_1 - \text{sech}\langle \beta_2 (\ln|\dot{\epsilon}^{in}| - \beta_3) \rangle \quad (17)$$

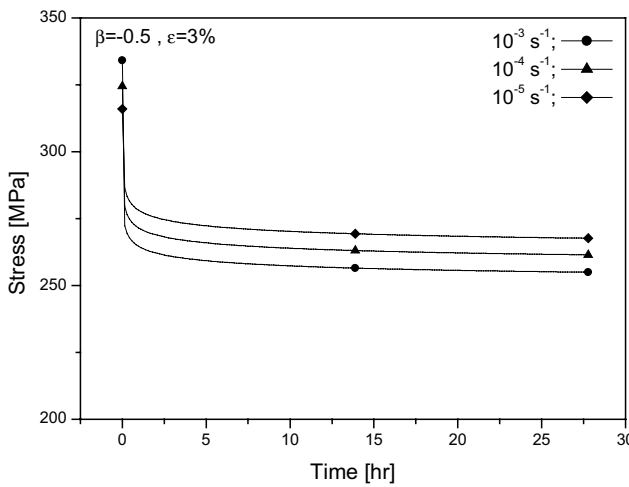
**Figure 5** : Relaxation behavior for  $\beta = 0.5$ . (a) Stress and equilibrium stress versus strain; (b) Stress at 3% strain versus time.

Negative rate sensitivity of flow stress occurs in a certain range of strain rates satisfying the condition,  $\beta < -1$ . The material constants  $\beta_2$  and  $\beta_3$  mainly control the extent and the position of the region in which there exist negative rate sensitivity, respectively. The capabilities are shown for the flow stress at 3% strain in Figs. 9 and 10.

Negative rate sensitivity as a manifestation of dynamic strain aging shows in general strain and strain rate dependence. In the most realistic case the augmentation function is thus made to depend on both the accumulated inelastic strain and the effective inelastic strain rate, see



(a)



(b)

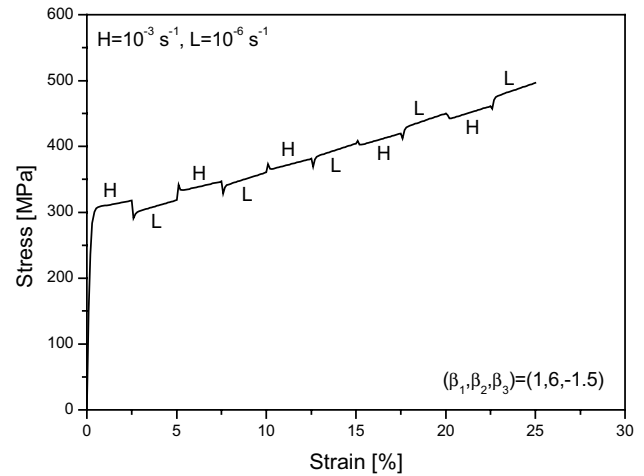
**Figure 6 :** Relaxation behavior for  $\beta = -0.5$ . (a) Stress and equilibrium stress versus strain; (b) Stress at 3% strain versus time.

Ho (1998).

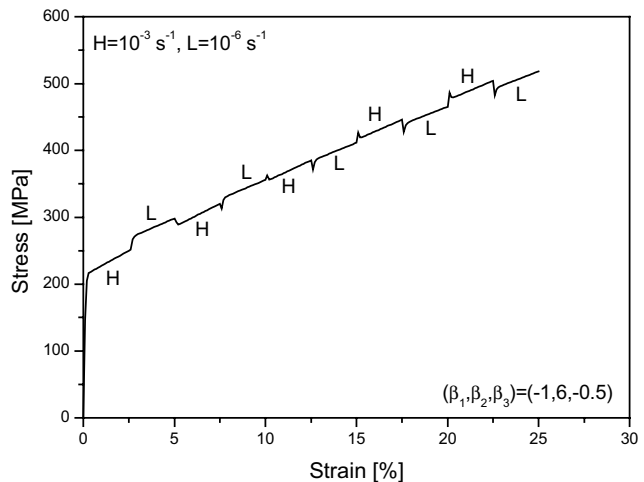
Some metals and alloys exhibit a significant increase of the flow stress at very high strain rates. This property can be incorporated in constitutive equations by decreasing the dynamic recovery effect in the evolution law for the equilibrium stress through the augmentation function given by

$$\beta = \beta_1 + (\beta_2 |\dot{\epsilon}^{in}|)^{\beta_3} \quad (18)$$

The point from which strain rate sensitivity abruptly in-



**Figure 7 :** The strain rate alternates between high ( $H = 10^{-3} \text{ s}^{-1}$ ) and low ( $L = 10^{-6} \text{ s}^{-1}$ ).  $\beta$  defined by equation (16) is used;  $\beta_1 = 1$ ,  $\beta_2 = 6$ ,  $\beta_3 = -1.5$ .



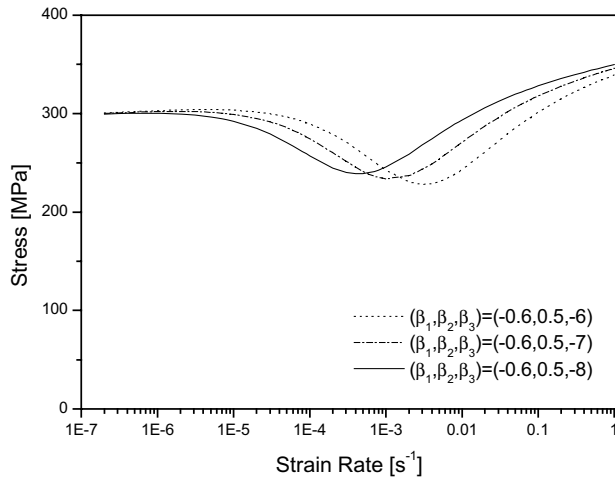
**Figure 8 :** The strain rate alternates between high ( $H = 10^{-3} \text{ s}^{-1}$ ) and low ( $L = 10^{-6} \text{ s}^{-1}$ ).  $\beta$  defined by equation (16) is used;  $\beta_1 = -1$ ,  $\beta_2 = 6$ ,  $\beta_3 = -0.5$ .

creases can be adjusted by appropriate values of the material constant  $\beta_2$  as shown in Fig. 11. Figure 12 shows how to control the slope of the strong increase.

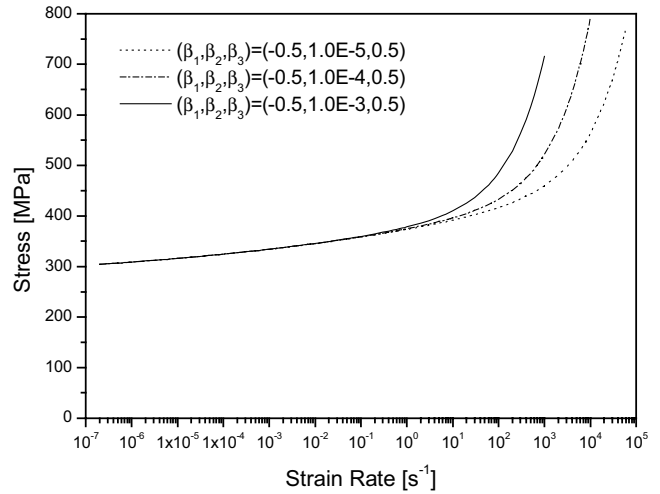
### 3.2 Modified 9Cr-1Mo steel

Experimental results of a modified 9Cr-1Mo steel reported by Yaguchi and Takahashi (1999) showed that the material exhibited positive rate sensitivity above  $500^\circ$ , and zero or negative rate sensitivity below  $400^\circ$  as a manifestation of dynamic strain aging. On the other

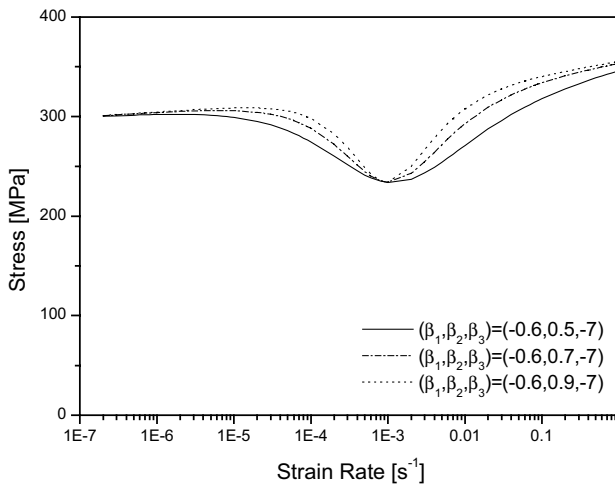




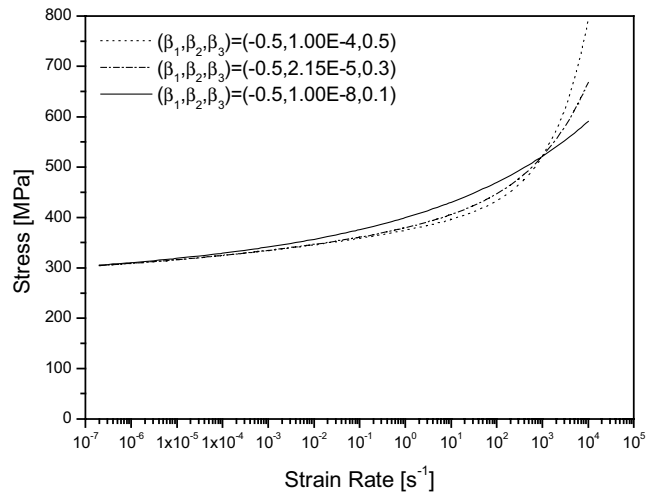
**Figure 9** : Stress at 3% strain versus strain rate.  $\beta$  defined by equation (17) is used.



**Figure 11** : Stress at 3% strain versus strain rate.  $\beta$  defined by equation (18) is used.



**Figure 10** : Stress at 3% strain versus strain rate.  $\beta$  defined by equation (17) is used.



**Figure 12** : Stress at 3% strain versus strain rate.  $\beta$  defined by equation (18) is used.

hand, the normal relaxation behavior was always observed at all test temperatures; i.e., all the time the magnitude of stress drop during the same relaxation periods increased with the prior strain rate. The constitutive equations are applied to model this deformation behavior with the help of the augmentation function.

Without the static recovery term in Eq. (8b), the theory may reproduce only “cold” creep behavior of materials. It is necessary to incorporate the effect of thermal diffusion at high homologous temperature into constitutive equations. To this end, VBO has been modified the evolution law for the equilibrium stress by the addition of the

static recovery term of which influence is only important at long test duration, see Choi (1989), Maciucescu et al. (1998) and Ho and Krempl (2001). It will be demonstrated that the observed relaxation behavior by Yaguchi and Takahashi (1999) are easily reproduced with the help of the classification criteria related to the value of the augmentation function.

To determine the material constants of the constitutive equations, we first estimate the equilibrium stress at a certain strain  $\epsilon_a$  in the flow stress region, which is assumed to satisfy the asymptotic solutions. When a ma-

material is loaded up to  $\varepsilon_a$  for a constant strain rate  $\dot{\varepsilon}_s$  and the strain is then held constant, the stress decays and approaches a saturated value that may be considered as the equilibrium stress at  $\varepsilon = \varepsilon_a$  for the strain rate  $\dot{\varepsilon}_s$ . It is evident from Eq. (8a) that the stress relaxation stops when the stress and the equilibrium stress are equal. Thus we can estimate the overstress at  $\varepsilon = \varepsilon_a$  for a constant strain rate  $\dot{\varepsilon}_s$  of monotonic tensile tests.

Once the overstress stress is known the augmentation function  $\beta$  can be calculated from

$$\beta = \frac{\{\sigma_H\} - \{\sigma_L\}}{\{\Gamma_H\} - \{\Gamma_L\}} - 1 \quad (19)$$

where the subscripts ‘‘H’’ and ‘‘L’’ denote the quantity at  $\varepsilon_a$  for high and low strain rate tests, respectively. To arrive at Eq. (19), Eq. (13) is used for the two different strain rates. In the previous version by Ho (1998) and Ho and Krempl (2001), it is not easy to estimate the overstress since the equilibrium stress changes significantly during relaxation period due to the stress rate term in the evolution law for the equilibrium stress. The previous version thus determines the value of the augmentation function by trial and error, whereas the present one estimates the overstress through relaxation test and then determines the augmentation function through Eq. (19).

Since  $\hat{E}_t$ , thus  $E_t$  is determined by measuring the slope of stress-strain curves the kinematic stress  $f$  at  $\varepsilon = \varepsilon_a$  can be approximately calculated by integrating Eq. (8c) with the help of Eq. (11). The value  $A_f = A_0$  for  $\dot{A} = 0$  is determined from Eq. (13) by first assuming a positive value for  $A_c$  that is introduced only to eliminate singularity in view of mathematics occurring in the case of  $A_f + \beta\Gamma = 0$ .

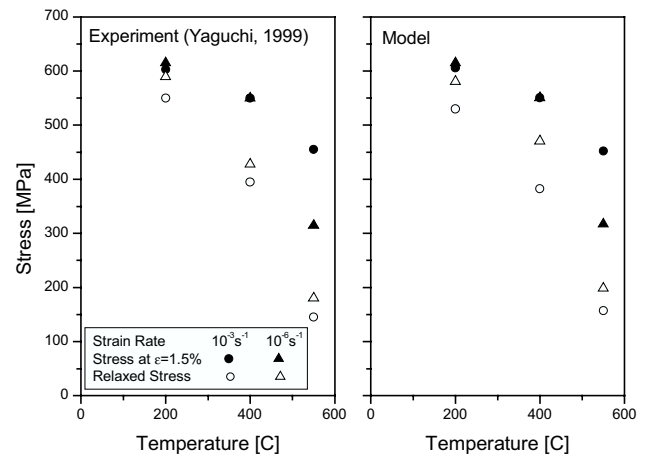
The material constants  $k_1$ ,  $k_2$  and  $k_3$  for the viscosity function are determined by the following procedure. Equation (11) is used to calculate the value of  $k[\Gamma]$  and Eq. (7) is rewritten in the form

$$\ln(k) = \ln(k_1) - k_3 \ln\left(1 + \frac{\Gamma}{k_2}\right) \quad (20)$$

With the experimental data of the tensile and the stress relaxation for at least three different strain rates, the three material constants can be determined by the means of least squares. The material constant  $\psi$  is determined by

variations to give best agreement with the transition between the initial quasi-elastic behavior and the full inelastic flow. Since some material constants are obtained with estimates it is necessary to compensate material constants so as to match the experimental data. The resulting set of the material constants is shown in Table 1.

Figure 13 shows that the flow stress at 1.5% strain for  $10^{-6}$  and  $10^{-3}s^{-1}$  strain rates exhibits slight negative, zero and positive rate sensitivity at  $200^\circ$ ,  $400^\circ$  and  $550^\circ$ , respectively. In performing the simulation at  $550^\circ$ ,  $A_r = 2.6 \times 10^{-6}s^{-1}$  and  $A_f = 100MPa$  are used since the evolution for the isotropic stress is needed with the static recovery effect in Eq. (8b) to incorporate the thermal diffusion. Even though the strain rate sensitivity is different at each temperature, the relaxed stress corresponding to the prior strain rate  $10^{-3}s^{-1}$  is always smaller than that of  $10^{-6}s^{-1}$  at the end of the 24-hr relaxation test. The comparison of the prediction of the model with the experimental data shows good correspondence.

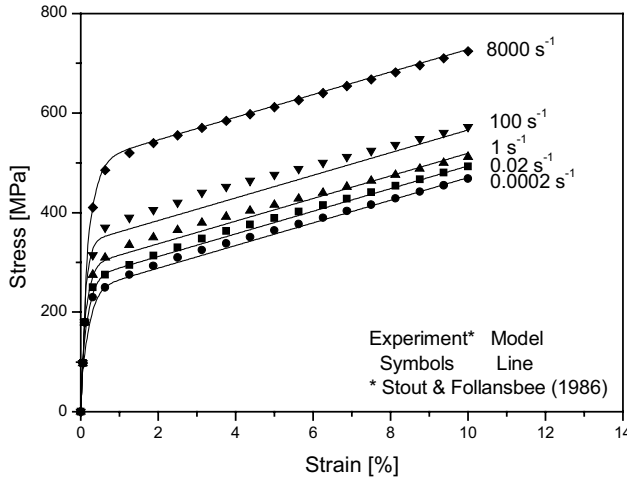


**Figure 13** : Stress at  $\varepsilon = 0.015$  and relaxed stress at the end of the 24-hr relaxation test.

### 3.3 304L stainless steel

Many researchers have studied the strong rate sensitivity of flow stress at very high strain rates, see for examples Clifton (1990) and Bodner and Rubin (1994). Using the augmentation function defined in Eq. (18), the constitutive equations incorporate the increasing value of the augmentation function ( $\beta \geq \beta_1$ ) with an increase of the effective inelastic strain rate. The increase in  $\beta$  reduces the effect of the dynamic recovery term in the evolution

law for the equilibrium stress, see Eqs. (8b) and (13). The property of power function, Eq. (18) causes a sharp increase of flow stress at very high strain rates.



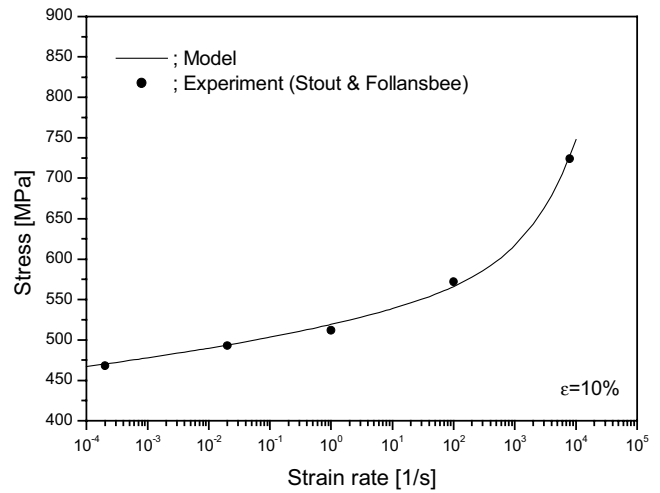
**Figure 14 :** Stress-strain curves for 304L stainless steel.  $\beta$  defined by equation (18) is used;  $\beta_1 = -0.5$ ,  $\beta_2 = 1.8 \times 10^{-5}$ ,  $\beta_3 = 0.5$ .

Figure 14 shows the comparison of the model using the material constants given in Table 1 with experimental results of the 304L stainless steel conducted by Stout and Follansbee (1986). Since the experimental compression tests by Stout and Follansbee were performed under isothermal conditions at strain rates less than  $0.1 \text{ s}^{-1}$  and adiabatic conditions at high strain rates, experimental stress-strain curves showed intersection between the curves. The model assumes only isothermal condition and the comparison in Fig. 14 is thus made up to 10% strain where the temperature rise can be neglected as mentioned by Stout and Follansbee.

A conventional method to represent the rate sensitivity of flow stress is to plot stress versus strain rate on a logarithm axis at a particular strain. The model reproduces the abrupt increase of rate sensitivity of the flow stress at 10% strain, which begins at the strain rate of about  $10^2 \text{ s}^{-1}$  as shown in Fig. 15.

#### 4 Discussion and conclusions

This generalized viscoplasticity theory, which has been conceived and developed by the author, is very competent to model all types of rate sensitivity in a consistent way. Positive, zero and negative rate sensitivity of the stress-



**Figure 15 :** Stress at  $\epsilon = 0.1$  versus strain rate for 304L stainless steel.  $\alpha$  defined by equation (18) is used;  $\beta_1 = -0.5$ ,  $\beta_2 = 1.8 \times 10^{-5}$ ,  $\beta_3 = 0.5$ .

strain curves are classified with respect to the augmentation function. In relaxation behavior the model predicts that the magnitude of the stress drop increases with an increase of prior strain rate and the relaxed stress associated with the fastest prior strain rate has the smallest magnitude regardless of the type of strain rate sensitivity. Using a proper augmentation function, the constitutive equations can reproduce zero and negative rate sensitivity depending on strain or strain rate, and an abrupt increase of rate sensitivity at very high strain rates.

As verification, the theory was applied to model a modified 9Cr-1Mo steel and 304L stainless steel and simulated quite well the observed mechanical responses.

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#### Appendix: Asymptotic Solutions

For the uniaxial case the following relations are obtained in the absence of the static recovery term, i.e.  $r = 0$  in Eq. (8b). If  $r \neq 0$  no asymptotic state is mathematically possible. However, it is believed that the relations approximately hold in the presence of a static recovery term. The integral representation of the constitutive equations is first considered to obtain asymptotic solutions for constant strain rate. Equations (8a), (8b) and (6) can be converted to

**Table 1** : Material Constants

	Hypothetical Material	Modified 9Cr-1Mo			304L
		200 °	400 °	550 °	
E[MPa]	200000	200000	190000	165000	195000
E <sub>t</sub> [MPa]	1000	4000	3000	500	2300
A <sub>c</sub> [MPa]	10	10	10	10	5
A <sub>0</sub> [MPa]	250	575	513	227	200
r[s <sup>-1</sup> ]	0	0	0	3.0E-7	0
β	Defined	-1.15	-1	-0.45	Defined
ψ[MPa]	300000	900000	600000	150000	100000
k <sub>1</sub> [s]	300000	200000	200000	200000	100000
k <sub>2</sub> [MPa]	70	60	180	250	100
k <sub>3</sub>	16	12	12	12	18

$$(\sigma - g) = (\sigma_0 - g_0) \exp \left[ - \int_{t_0}^t \frac{1}{k} d\zeta \right] + \int_{t_0}^t E \left( \dot{\varepsilon} - \frac{\dot{g}}{E} \right) \exp \left[ - \int_{\zeta}^t \frac{1}{k} ds \right] d\zeta \quad (A-1)$$

$$\{A\} = A_f \quad (A-6)$$

where brace designates the value of the quantity at  $t \rightarrow \infty$ . By differentiation of Eqs. (A-1), (A-2) and (A-3) and taking the limit for  $t \rightarrow \infty$ , we obtain Eqs. (9) and (10).

Using Eqs. (8a), (8c) and (9), the inelastic strain rate in the asymptotic state leads to

$$(g - f) = (g_0 - f_0) \exp \left[ - \int_{t_0}^t \frac{\psi |\dot{\varepsilon}^{in}|}{A_c + |A + \beta \Gamma|} d\zeta \right] + \int_{t_0}^t \psi \dot{\varepsilon}^{in} \exp \left[ - \int_{\zeta}^t \frac{\psi |\dot{\varepsilon}^{in}|}{A_c + |A + \beta \Gamma|} ds \right] d\zeta \quad (A-2)$$

$$\{\dot{\varepsilon}^{in}\} = \frac{1}{1 + (E_t/E)} \dot{\varepsilon} \quad (A-7)$$

It can be seen from Eqs. (8c) and (A-7) that the kinematic stress is rate-independent in the asymptotic state. With the help of Eqs. (8a), (9) and (A-7), Eqs. (11) and (12) are obtained from Eqs. (A-4) and (A-5), respectively.

$$A = A_f + (A_0 - A_f) \exp \left[ - \int_{t_0}^t A_r p d\zeta \right] \quad (A-3)$$

where the subscript zero denotes the initial value of the subscripted quantity at  $t = t_0$ . The above equations for infinite time limits can be written as

$$\{\sigma - g\} = \left( E - \left\{ \frac{dg}{d\varepsilon} \right\} \right) \{k\} \dot{\varepsilon} \quad (A-4)$$

$$\{g - f\} = \left( 1 - \frac{1}{E} \left\{ \frac{df}{d\varepsilon} \right\} \right) \left\{ \frac{A_c + |A_f + \beta \Gamma|}{|\dot{\varepsilon}^{in}|} \right\} \dot{\varepsilon} \quad (A-5)$$

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