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Optimum Design of Adaptive Truss Structures Using the Integrated Force Method

R. Sedaghati, A. Suleman[1], S. Dost and B. Tabarrok[2]

Abstract: A structural analysis and optimization method is developed to find the optimal topology of adaptive determinate truss structures under various impact loading conditions. The objective function is based on the maximization of the structural strength subject to geometric constraints. The dynamic structural analysis is based on the integrated finite element force method and the optimization procedure is based on the Sequential Quadratic Programming (SQP) method. The equilibrium matrix is generated automatically through the finite element analysis and the compatibility matrix is obtained directly using the displacement-deformation relations and the Single Value Decomposition (SVD) technique. By combining the equilibrium and the compatibility matrices with the force-displacement relations, the equations of the motion are obtained with element force as variables. The proposed method is extremely efficient to analyze and optimize adaptive truss structures. It is observed that the structural strength is improved significantly using the adaptively optimized geometries while the computational effort required by the force method is found to be significantly lower than that of the displacement method.

keyword: Adaptive Structures, Force Method, Topology Optimization Dynamic Analysis

1 Introduction

The concept of equilibrium of forces and compatibility of deformations is fundamental to analysis methods for solving problems in structural mechanics. The equilibrium equations need to be augmented by the compatibility conditions since the equilibrium equations are indeterminate by nature, and determinacy is achieved by adding the compatibility conditions. Generally, two analytical methods (displacement and force) are available to analyze determinate and indeterminate structures.

Structural analysis and optimization algorithms developed in recent years have generally been based on the displacement method [Venkayya(1978); Canfield, Grandhi and Venkayya(1988); Mohr(1992); Mohr(1994); Flurry and Schmit(1980); Haftka and Gurdal(1992)]. The displacement method is an efficient approach for stress-displacement type analysis, however it presents disadvantages in optimization problems when the number of stress constraints are larger than the displacement constraints, or in geometry optimization where the element forces are the main primary variables. For a *determinate structure* or a *not highly redundant structure* (the number of redundant elements is lower than the displacement degrees of freedom), analysis using the force method is computationally more efficient than the displacement method. However, the force method has not been very popular among researchers in structural optimization problems because the redundancy analysis required in the force method has not been amenable to computer automation.

In the classical form of the force method, it is very difficult to generate the compatibility conditions. Splitting the given structure into a determinate basis structure and redundant members generates the compatibility in the classical force method. The compatibility conditions are written by establishing the continuity of deformations between redundant members and the basis structure. Navier [Timoshenko(1953)] originally developed this procedure for the analysis of indeterminate trusses. Prior to the 1960s, the basis structure and redundant members were generated manually. In the post-1960s, several schemes have been devised to automatically generate redundant members and the basis determinate structure [Robinson(1965); Kaneko, Lawo, and Thierauf(1982)], however with limited success. In the integrated force method developed by Patnaik [Patnaik(1986); Patnaik and Joseph(1986); Patnaik, Berke, and Gallagher(1991)] both equilibrium equations and

[1] Associate Professor, IDMEC.
[2] Department of Mechanical Engineering
University of Victoria, B.C.
Canada V8W 3P6
e-mail: rsedagha@me.uvic.ca

compatibility conditions are solved simultaneously. The generation of compatibility equations is based on extending St. Venant's theory of elasticity strain formulation to discrete structural mechanics and eliminating the displacements in the deformation-displacement relation.

The application of the force method based on the minimization of the complementary energy to analyze the topology optimization of adaptive truss structures under static loading has been reported by Sedaghati, Tabarrok, Suleman and Dost(2000). In the present study, the integrated force method has been used to analyze and optimize adaptive truss structures under static and dynamic loading [Sedaghati, Tabarrok and Suleman(2000)]. The equilibrium and compatibility equations are solved simultaneously. A direct method has been developed to generate the compatibility matrix for indeterminate truss structures. The method is based on the displacement-deformation relation and singular value decomposition (SVD) technique and there is no need to select consistent redundant members. For determinate truss structures the equilibrium matrix is generated automatically through the finite element analysis.

For optimum structural design, the design variables are selected so as to minimize/maximize the objective function while satisfying the required constraints. The constraints may include allowable stresses in the elements, limitation on geometric parameters, displacement limits at the joints, frequency specifications, system stability, etc. Depending on the nature of the applied loads, the structure and its geometry, one or more of the constraints can be active and control the design of the structure. For the geometry optimization of the adaptive truss structures, the structural strength is selected as the objective function and constraint equations are based on geometry parameters (such as angles) of the active members.

The application and efficiency of the proposed method is illustrated by optimizing the topology of an adaptive truss structure using active elements in order to maintain maximum structural strength under various impact loading conditions. The force method has proved to be perfectly natural and efficient because the primary variables in the topology optimization of adaptive truss structures are the member forces.

2 Structural Analysis Using the Force Method

A discrete finite element structure can be designated as structure (d, f), where d and f are the displacement and force degrees of freedom, respectively. The structure (d, f) has d equilibrium equations and $r = (f - d)$ compatibility conditions. In static problems the equilibrium equations in the displacement formulation can be written as

$$\mathbf{KU} = \mathbf{P} \tag{1}$$

where $\boldsymbol{K}$ is the system stiffness matrix of the structure (obtained by assembling the stiffness matrices of the individual elements); $\mathbf{P}$ is the external applied load vector; and $\mathbf{U}$ is the nodal displacement vector. The compatibility conditions have been satisfied implicitly during the generation of the Eq. (1). The equivalent form of the Eq. (1) in the integrated force formulation can be written as [Patnaik(1986)]:

$$\mathbf{SF} = \mathbf{P}^* \tag{2}$$

where $\mathbf{F}$ is the element force vector. The matrix $\boldsymbol{S}$ and vector P^* can be obtained through combination of the equilibrium matrix as

$$\mathbf{QF} = \mathbf{P} \tag{3}$$

and compatibility equations as

$$\mathbf{C}\Delta = \mathbf{0} \tag{4}$$

where element deformation vector, Δ, can be related to the element force vector, $\mathbf{F}$, according to

$$\Delta = \mathbf{GF} \tag{5}$$

thus

$$\mathbf{S} = \begin{bmatrix} \mathbf{Q} \\ \cdots \\ \mathbf{C}\ \mathbf{G} \end{bmatrix}, \quad \mathbf{P}^* = \begin{bmatrix} \mathbf{P} \\ \cdots \\ \mathbf{0} \end{bmatrix} \tag{6}$$

where $\boldsymbol{Q}$, $\boldsymbol{C}$ and $\boldsymbol{G}$ are the $(d \times f)$ equilibrium matrix, $(r \times f)$ compatibility matrix and $(f \times f)$ flexibility matrix, respectively. The matrices $\boldsymbol{Q}$, $\boldsymbol{C}$ and $\boldsymbol{G}$ are banded and they have full-row ranks of d, r and f, respectively and the matrices $\boldsymbol{Q}$ and $\boldsymbol{C}$ depend on the geometry of the structure and are independent of material properties. For a finite element idealization, the generation of the equilibrium matrix $\boldsymbol{Q}$ and the flexibility matrix $\boldsymbol{G}$ is straightforward and can be obtained automatically. However the automatic generation of the compatibility matrix $\boldsymbol{C}$ is a laborious task in the standard force method. In the integrated force method, the generation of $\boldsymbol{C}$ is based on the elimination of the d displacement degrees of freedom from f elemental deformations. Here, an efficient method is proposed to derive the compatibility matrix directly. The method is based on the displacement-deformation relations and SVD.

The displacement-deformation relation for discrete structures can be obtained by equating internal strain energy and external work as

$$\frac{1}{2}\mathbf{F}^T\Delta = \frac{1}{2}\mathbf{P}^T\mathbf{U} \tag{7}$$

By substituting $\mathbf{P}$ from Eq. (3) into Eq. (7), we can obtain

$$\frac{1}{2}\mathbf{F}^T\mathbf{Q}^T\mathbf{U} = \frac{1}{2}\mathbf{F}^T\Delta \quad \text{or} \quad \mathbf{F}^T(\mathbf{Q}^T\mathbf{U}-\Delta) = 0 \tag{8}$$

Since the element force vector $\mathbf{F}$ is not a null space, we finally obtain the following relation between member deformation vector and nodal displacement vector

$$\Delta = \mathbf{Q}^T\mathbf{U} \tag{9}$$

Equation (9) relates the f deformations to the d nodal displacement degrees of freedom; therefore the $r = (f-d)$ compatibility equations can be obtained through elimination of the d nodal displacements from the f deformations. To obtain the compatibility matrix, we may express nodal displacements in terms of member deformations using Eq. (9) as

$$\mathbf{U} = \left(\mathbf{Q}\mathbf{Q}^T\right)^{-1}\mathbf{Q}\Delta = \left(\mathbf{Q}^T\right)^{pinv}\Delta \tag{10}$$

where the matrix $\left(\mathbf{Q}^T\right)^{pinv}$ denotes the Moore-Penrose pseudo-inverse of $\mathbf{Q}^T$. Considering Eq. (9) and (10), we may have

$$\left[\mathbf{I}-\mathbf{Q}^T\left(\mathbf{Q}^T\right)^{pinv}\right]\Delta = 0 \tag{11}$$

or

$$\mathbf{A}\Delta = \mathbf{0} \tag{12}$$

where

$$\mathbf{A} = \left[\mathbf{I}-\mathbf{Q}^T\left(\mathbf{Q}^T\right)^{pinv}\right] \tag{13}$$

Equation (12) is similar to the compatibility equations in Eq. (4), however matrix $\boldsymbol{A}$ is a $(f\times f)$ matrix with rank of r. It means that the rows of matrix $\boldsymbol{A}$ are dependent on each other. In order to extract the $(r\times f)$ compatibility matrix $\boldsymbol{C}$ from the matrix $\boldsymbol{A}$, i.e. to reduce the matrix $\boldsymbol{A}$ to matrix $\boldsymbol{C}$, the singular value decomposition (SVD) method is used [Golub and Van Loan(1996)]. Applying SVD to $\boldsymbol{A}$, we obtain

$$\mathbf{A} = \mathbf{R}\Sigma\mathbf{T}^T \tag{14}$$

where $\boldsymbol{R}$ and $\boldsymbol{T}$ are $(f\times f)$ orthogonal matrices and

$$\Sigma = \begin{bmatrix} \Lambda & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{(f\times f)} \tag{15}$$

with $\Lambda = \mathrm{diag}\{\sigma_1\ \ \sigma_2\ \ \ldots\sigma_r\}$, and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$. It follows that

$$\mathbf{A} = \mathbf{R}\begin{bmatrix} \mathbf{C} \\ 0 \end{bmatrix} \tag{16}$$

Therefore the $(r\times f)$ compatibility matrix $\boldsymbol{C}$ can be represented as

$$\mathbf{C} = \Lambda[\mathbf{T}_1\ \ \mathbf{T}_2\ \ \ldots\ \ \mathbf{T}_i\ \ \ldots\ \ \mathbf{T}_r]^T \tag{17}$$

where the vector $\mathbf{T}_i$ denotes the i^{th}column of matrix $\boldsymbol{T}$.

It is noted that SVD is not a cheap technique, thus for high dimension structures or highly redundant structures the advantage of the integrated force method based on the SVD method may be reduced or even reversed.

Although Eq. (9) is sufficient to obtain the element deformations using nodal displacements, it is not sufficient to obtain nodal displacements using element deformations or forces because redundant structures are represented by rectangular equilibrium matrix Q with no inverse. This implies that the compatibility equations should be merged with the equilibrium equations. For this reason, using $\mathbf{S}$ instead of $\mathbf{Q}$ in Eq. (9) and solving for nodal displacements $\mathbf{U}$, we obtain

$$\mathbf{U} = \mathbf{J}\,\Delta \quad \text{or} \quad \mathbf{U} = \mathbf{J}\,\mathbf{G}\,\mathbf{F} \tag{18}$$

where

$$\mathbf{J} = d \quad \text{rows of} \quad \mathbf{S}^{-T} \tag{19}$$

In dynamic problems the equations of the motion in the displacement formulation can be written as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{P} \tag{20}$$

where $\boldsymbol{M}$ is the stiffness matrix of system and $\ddot{\mathbf{U}}$ is acceleration vector.

Considering Eq. (18) and noting that $\mathbf{KU}$ in the displacement method is equivalent to $\mathbf{S\,F}$ in the force method, Eq. (21) may be written as

$$\mathbf{M}^*\ddot{\mathbf{F}}+\mathbf{SF}=\mathbf{P}^* \tag{21}$$

where

$$\mathbf{M}^*=\begin{bmatrix}\mathbf{M\,J\,G}\\ \cdots\cdots\\ \mathbf{0}\end{bmatrix} \tag{22}$$

Eq. (21) represents the equations of the motion in the framework of the force formulation.

3 Numerical Integration

In this analysis, the Newmark direct integration method [Newmark(1959); Cook, Malkus and Plesha(1989)] is used to solve Eq. (21). It is assumed that the initial values of force vector $\mathbf{F}$ and the vector $\dot{\mathbf{F}}$ at time t=0 are known. The vector $\ddot{\mathbf{F}}$ at t=0 is obtained from Eq. (21). Considering a time increment Δt, the predictor parameters, $\hat{\mathbf{F}}_{n+1}$ and $\hat{\dot{\mathbf{F}}}_{n+1}$ at time $(n+1)\ \Delta t$ in terms of the known vectors at time $n\ \Delta t$ are computed as

$$\begin{aligned}\hat{\mathbf{F}}_{n+1}&=\mathbf{F}_n+\Delta t\dot{\mathbf{F}}_n+0.5\Delta t^2(1-2\beta)\ddot{\mathbf{F}}_n\\ \hat{\dot{\mathbf{F}}}_{n+1}&=\dot{\mathbf{F}}+\Delta t(1-\gamma)\ddot{\mathbf{F}}_n\end{aligned} \tag{23}$$

Now, the vector $\ddot{\mathbf{F}}$ at time $(n+1)\ \Delta t$, $\ddot{\mathbf{F}}_{n+1}$ is obtained from the following equation:

$$\left(\mathbf{M}^*+\beta\Delta t^2\mathbf{S}\right)\ddot{\mathbf{F}}_{n+1}=\mathbf{P}^*_{n+1}-\mathbf{S}\hat{\mathbf{F}}_{n+1} \tag{24}$$

Knowing $\ddot{\mathbf{F}}_{n+1}$, the force vector $\mathbf{F}$ and the vector $\dot{\mathbf{F}}$ at time $(n+1)\ \Delta t$ are obtained from the following relations:

$$\begin{aligned}\mathbf{F}_{n+1}&=\hat{\mathbf{F}}_{n+1}+\beta\Delta t^2\ddot{\mathbf{F}}_{n+1}\\ \dot{\mathbf{F}}_{n+1}&=\hat{\dot{\mathbf{F}}}_{n+1}+\gamma\Delta t\ddot{\mathbf{F}}_{n+1}\end{aligned} \tag{25}$$

Constants β and γ in the above equations are the accuracy and stability parameters in the Newmark method. The Newmark method is unconditionally stable [Belytschoko and Huges(1983)] if

$$0.5\leq\gamma\leq2\beta \tag{26}$$

4 Geometry Optimization

An adaptive truss structure, which can change its configuration to maintain its structural strength by lengthening or shortening some of active member's length, is considered for geometry optimization in this study. All recent work in the optimum design of adaptive structures [Murotsu and Shao(1990a); Murotsu and Shao(1990b); Murotsu and Shao(1990c)] is based on the conventional displacement method. When optimizing the configuration of the adaptive truss structure, the objective is to maximize the structural strength [Murotsu and Shao(1990a); Murotsu and Shao(1990b); Murotsu and Shao(1990c)], by changing the lengths or angles of the active members. Here, it is assumed that the truss structure is composed of f members including f_a active members. The vector $\varphi=\left\{\varphi_1,\varphi_2,\ldots,\varphi_{f_a}\right\}$ includes the angles of the f_a active members as shown in the Figure 1 and these are selected as the design variables in order to be consistent with Refs. [Murotsu and Shao(1990a); Murotsu and Shao(1990b); Murotsu and Shao(1990c)]. The geometry optimization problem for adaptive truss

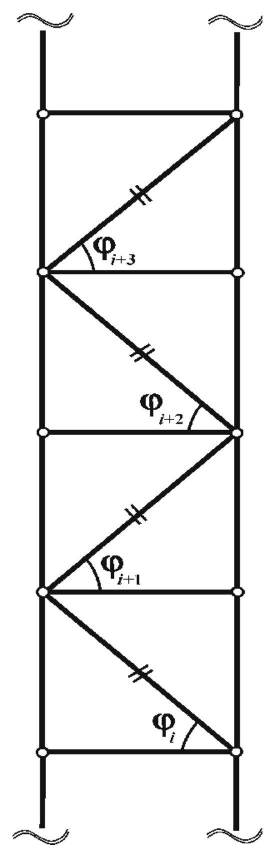

Figure 1 : Adaptive truss structure with active members.

structure under dynamic load may be defined mathematically as:

For a given external load $\mathbf{P}(t)$ and its direction ψ find the vector φ such that

$$\begin{array}{lll} \text{Minimize} & \text{M=Maximum}|\mathbf{F}_n(\varphi,\psi)| & \\ & & 1 \le n \le N \\ \text{Subject to} & g_j(\varphi) \le 0 & 1 \le j \le J \end{array} \quad (27)$$

where $\mathbf{F_n}$ is the element force vector at time step $n\,\Delta t$, g_j are geometrical constraints on active structural members and j and N are total number of geometrical constraints and total number of time steps.

For linear structural analysis the structural strength may be defined as follows:

$$S_s = \text{Maximum}\left|\frac{\overline{\mathbf{F}}}{\hat{\mathbf{F}}(\varphi,\psi)}\right| \quad (28)$$

for static load and,

$$S_d = \text{Maximum}\left|\frac{\overline{\mathbf{F}}}{\hat{\mathbf{F}}_{\mathbf{n}}(\varphi,\psi)}\right| \quad 1 \le n \le N \quad (29)$$

for dynamic load. Where $\overline{\mathbf{F}}$ is the vector of element strength including allowable element forces, $\hat{\mathbf{F}}$ is the element load effect vector due to unit external static load and $\hat{\mathbf{F}}_n$is defined as the element load effect due to unit external impact load.

Considering Eq. (29), it is noted that minimization of the maximum forces in Eq. (27) is equivalent to the maximization of the structural strength, S_d.

In this study, the Sequential Quadratic Programming (SQP) method has been applied to solve the optimization problem discussed. The implementation of the SQP has been done in MATLAB [Coleman, Branch and Grace(1999)]. Details of the SQP algorithm may be found in Powell(1978).

The objective function of Eq. (27) is not a smooth and convex function, thus local optimum result may be achieved using the gradient-based algorithms such as SQP algorithm. In this study, several randomly generated initial points have been used for the SQP algorithm to make sure that the optimal solution is global or very close to global solution.

5 Illustrative Example

The twenty-four-bar adaptive truss shown in Figure 2 has been optimized topologically to obtain the maximum structural strength in the presence of various static and dynamic loading conditions [Murotsu and Shao(1990a)]. It has four bays and every bay contains one active member. The truss consists of 24 members with active members 5, 9, 13 and 17. Angles $\varphi_1, \varphi_2, \varphi_3$ and φ_4 are taken as geometrical design variables , and these are related to the 4 active member's length. The ranges for those angles are specified as $0° \le \varphi_1, \varphi_2, \varphi_3, \varphi_4 \le 90°$. The material properties are: Young's modulus E=7×10^{10} N/m^2, Element strength $\overline{F}$=10000 N for all members. The geometrical parameters are: cross-sectional area A=10^{-4} m^2 and

L=2 m.

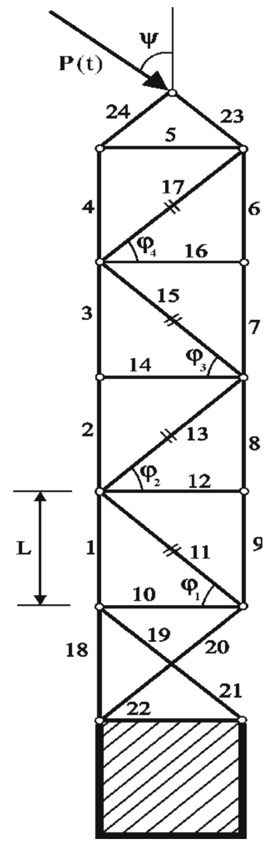

Figure 2 : The 24-bar plane adaptive truss structure with active members 11, 13, 15, and 17

The minimum and maximum natural frequency of the structure are $\omega_{min} = 8.2815$ Hz and $\omega_{max} = 1026.5$ Hz, respectively. For static load an external static load, P, is applied at node 1 and for dynamic load, an external impact load $P(t)$ is applied at node 1, and the following two cases are considered:

$$\text{Case I:} \quad P(t) = \begin{cases} I_p \,/\, \Delta t \text{ (N)} & 0 \le t \le \Delta t \\ 0 & t > \Delta t \end{cases}$$

$$\text{Case II:} \quad P(t) = \begin{cases} I_p \,/\, (100\Delta t) \text{ (N)} & 0 \le t \le 100\Delta t \\ 0 & t > 100\Delta t \end{cases}$$

where, $I_p = \sum P(t)\,\Delta t$ is applied impact for each case. In each case, the time increment Δt=1.3×10^{-4} sec and a total numerical time of $1000\times\Delta t$ =0.13 sec are considered.

First the adaptive structure is optimized under static load. The structural strength for the fixed configuration is plotted as the solid curve in Figure 3. The structural strength for the optimal adaptive geometries is plotted as the dashed curve. The optimal geometries of the adaptive structure are shown in Figure 4. The optimization results for the varying load direction ψ are shown in Table 1. The results indicate that the optimal adaptive truss structure maintains a high structural strength in comparison to the conventional structure by making the structure align itself with the external applied load direction. The computational time required by the force method was found to be about 5 times lower than that of displacement method for each load direction. The force method clearly provides a computationally more efficient and faster solution. The results are in excellent agreement with those in [Murotsu and Shao(1990a)].

Table 1 : Optimal values of $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and structural strength (N) in the optimal adaptive shapes for static load.

ψ°	Design Variables				Structural
	φ_1°	φ_2°	φ_3°	φ_4°	Strength(N)
-90	78.89	8.54	83.82	4.66	244.1
-80	80.70	6.81	83.16	6.02	346.7
-70	80.90	9.16	80.71	9.71	518.7
-60	76.98	13.43	76.76	14.82	724.2
-50	71.72	18.60	71.72	19.15	880
-40	66.79	23.21	66.79	23.32	1003.8
-30	63.36	26.64	63.36	26.64	1035.3
-20	55.03	34.97	55.03	34.97	1103.4
-10	48.75	41.24	48.76	41.24	1220.8
0	45	45	45	45	1414.2
10	38.31	51.69	38.31	51.69	1220.8
20	32.30	57.70	32.30	57.70	1103.2
30	25.96	64.10	25.90	64.10	1035.3
40	25.49	71.57	18.43	71.57	1003.8
50	20.66	77.60	13.48	77.91	948.5
60	11.89	80.02	11.49	78.62	694.6
70	9.12	82.83	9.07	80.84	508.9
80	10.23	84.74	5.14	83.10	276.7
90	20.43	83.33	7.11	86.22	208.3

For dynamic load in case I, the impact time on the structure is very short. It was observed that members 23 or 24 always produced the largest element forces, independently of the direction of the impact load $P(t)$, but if the external load is time-independent (static), member 18 in the bottom of the structure becomes the most critical. Members 23 and 24 are at the top of the structure where the direct impact load happens. Figure 5 shows the time-history of element forces in members 24 and 18 for the impact $I_p = 1$ N-sec and load direction of $\psi = 90°$. The results reveal that for stability parameters $\beta = 0.25$ and $\gamma = 0.5$, the numerical solution displays spurious "beating" in which the amplitude of response repeatedly grows and decays. The Newmark method, with the selected stability parameters (average acceleration algorithm or trapezoidal rule) is unconditionally stable and it does not have artificial numerical damping, which minimizes numerical noise. Taking $\gamma > 0.5$ introduces artificial damping, which automatically dissipates the high-frequencies noises.

In order to maximize the high-frequency dissipation for a given value of $\gamma > 0.5$, we have taken $\beta = 0.25\,(\gamma+0.5)^2$ introduced by Hughes(1987). Using $\gamma = 0.67$ and $\beta = 0.25\,(\gamma+0.5)^2 = 0.34$, it is obvious from the Figure 5 that high-frequency noise is clearly eliminated. The optimization algorithm introduced in Section 4 was used to find the optimal geometries for this case. However, no optimal geometries were found because the largest internal forces are always present in members 23 and 24. This means that for impact load acting in a very short time, the structural strength (allowable applied impact) is not affected by the geometry of the structure. The required CPU time to generate the time-history of element forces through the force method was 1.7 sec in comparison to the 32 sec using the displacement method.

For case study II, the time taken for the impact load to act on the structure has been considerably increased.

Figure 6 shows the time history of the element forces in members 24 and 18 for the impact $I_p = 1$ N-sec and load direction of $\psi = 90°$. It is observed that high-frequency noise is present for the stability parameters of $\gamma = 0.5$ and $\beta = 0.25$, but the numerical noise is eliminated by using $\gamma > 0.5$ and $\beta > 0.25$. It is interesting to note that

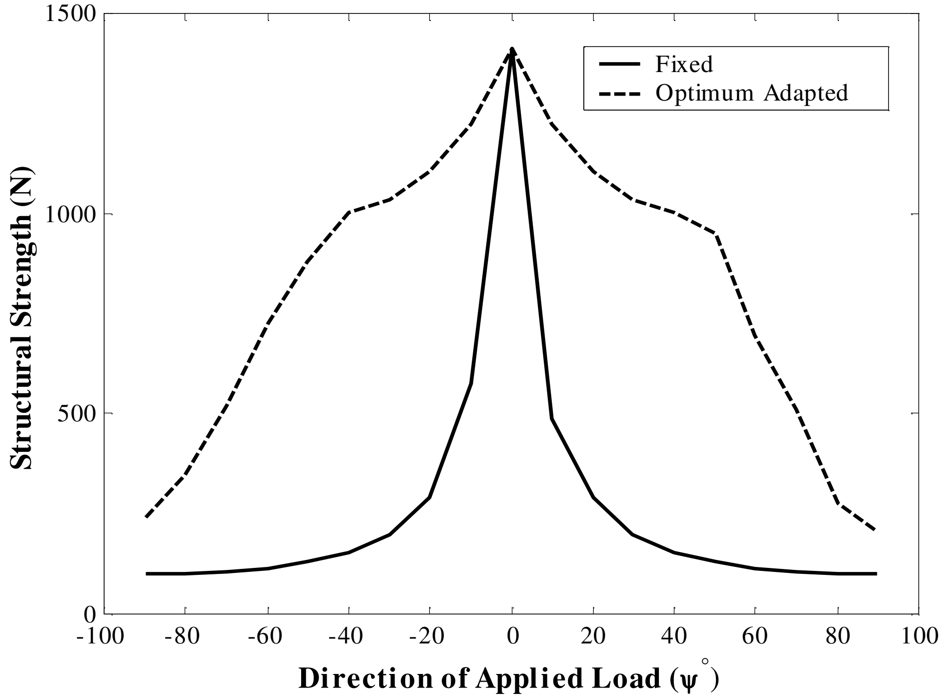

Figure 3 : Structural strength, S_s, versus the direction of the applied load for the fixed and optimized adapted structure under Static load.

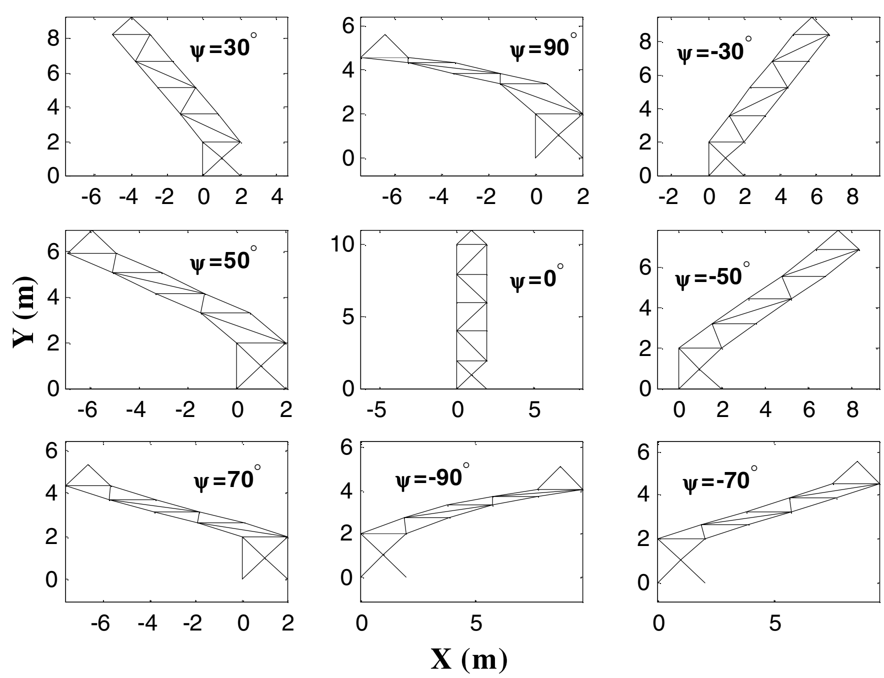

Figure 4 : Optimal adaptive shapes for static load.

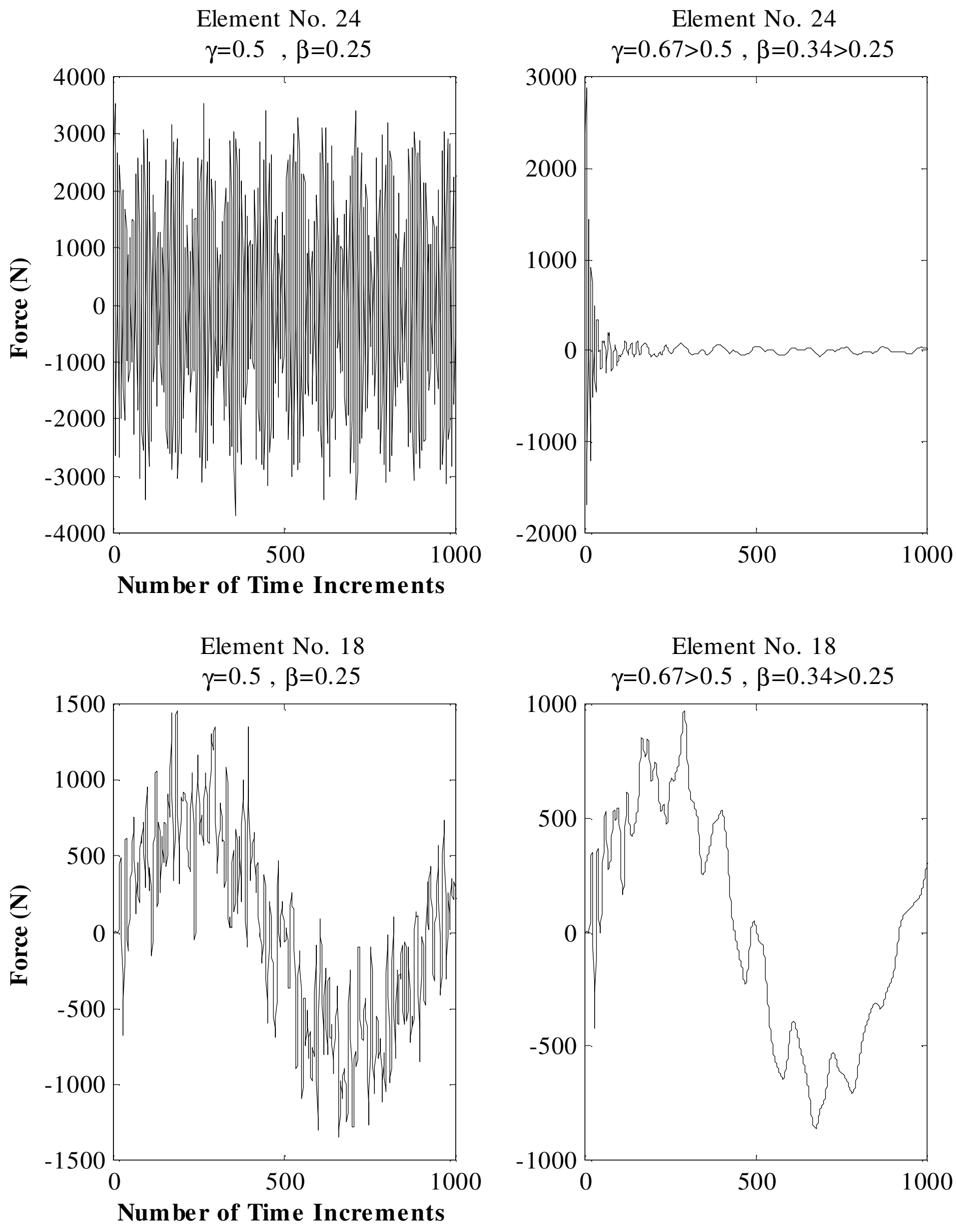

Figure 5 : Time histories of the element forces for case I and impact I_p = 1 N-sec.

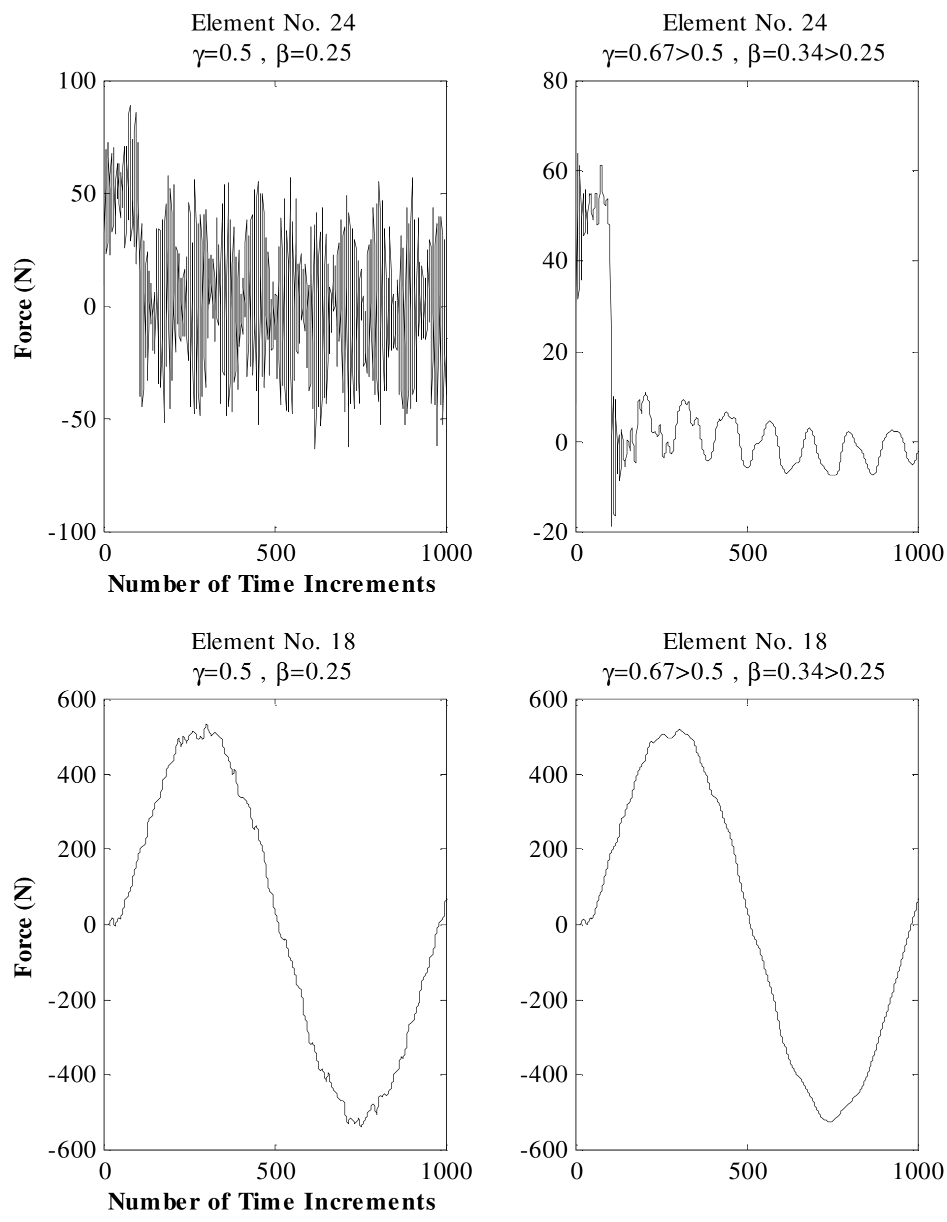

Figure 6 : Time histories of the element forces for case II and impact I_p = 1 N-sec.

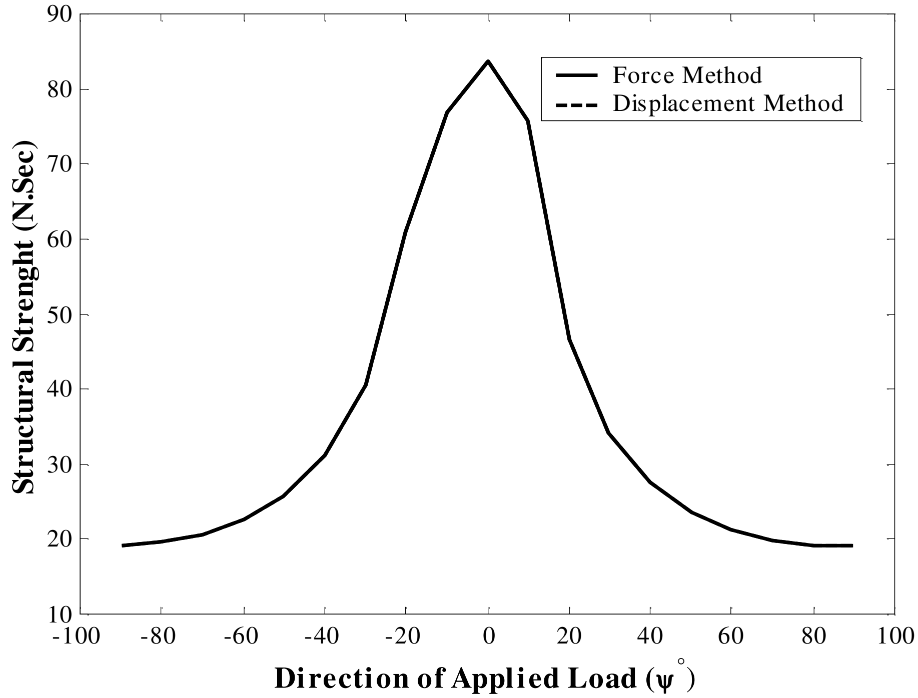

Figure 7 : Structural strength, S_d, versus the direction of the applied load for the fixed structure (Dynamic load).

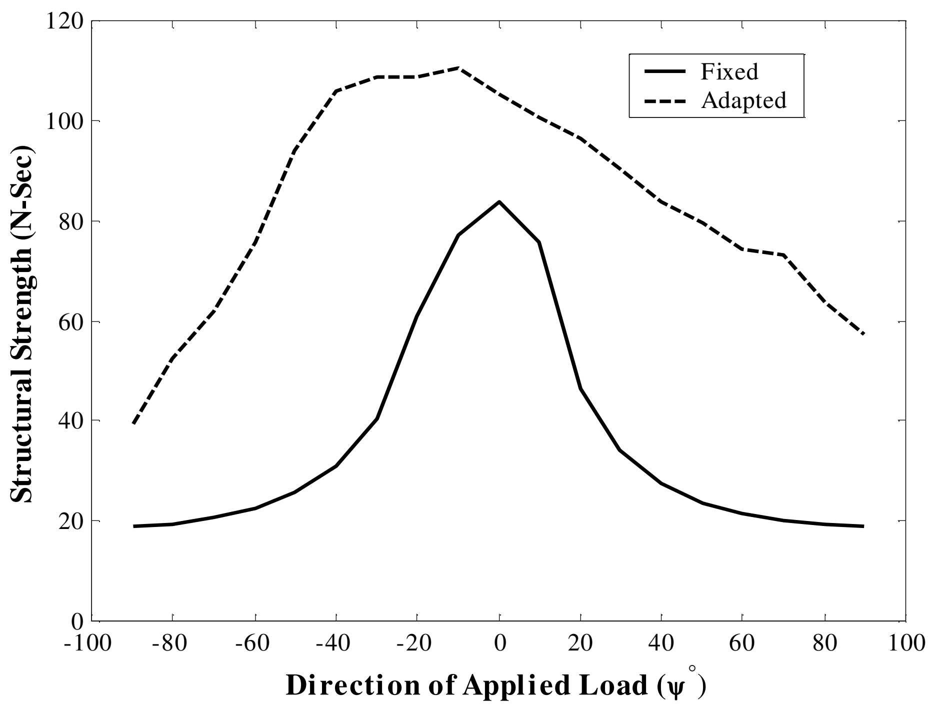

Figure 8 : Optimal adaptive and fixed structural strength versus the direction of the applied load (Dynamic load).

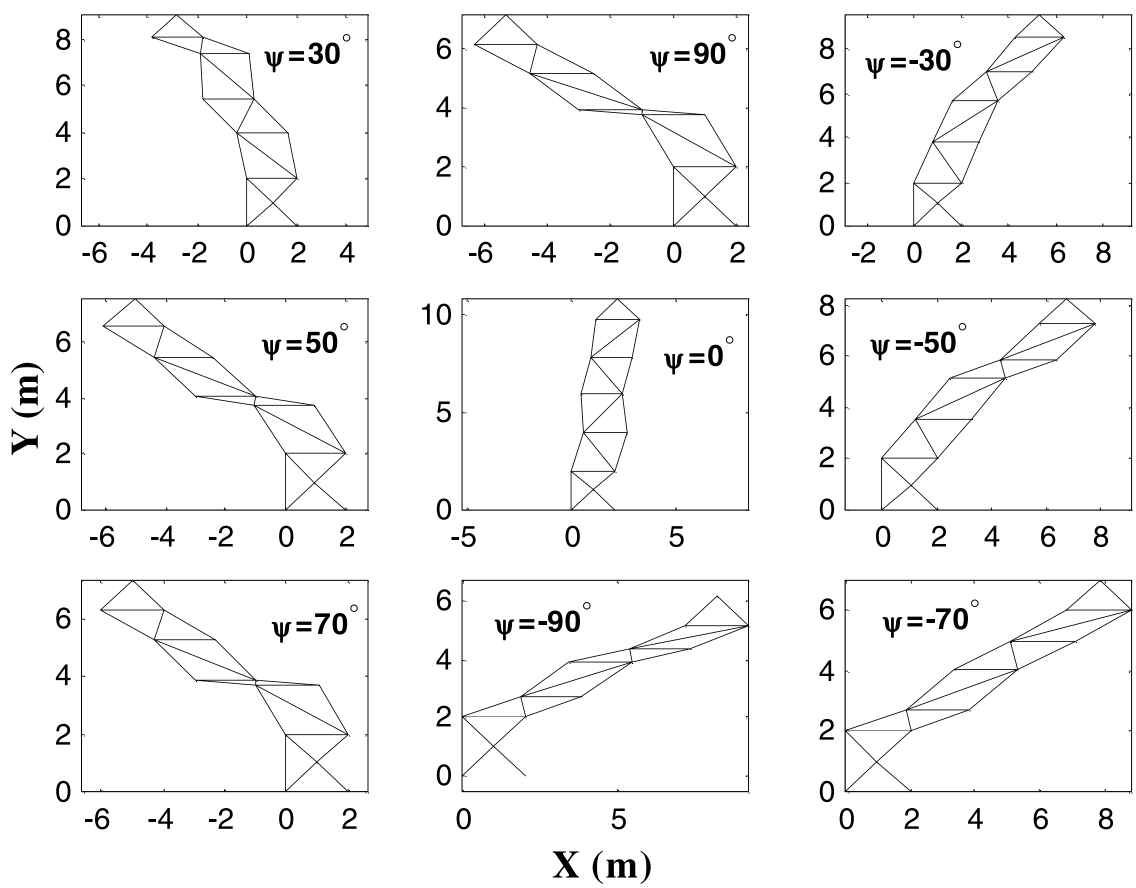

Figure 9 : Optimal adaptive shapes for dynamic load

because the impact load acts over a relatively large time in comparison to the Case I, the effect of high-frequency noise is not as noticeable as in the Case I, especially for member No. 18. The structural strength for the fixed configuration ($\varphi_1 = \varphi_2 = \varphi_3 \quad = \varphi_4$) for both displacement and force methods are shown in the Figure 7 and the results from both methods totally match, demonstrating the accuracy of the proposed method. The structure becomes very rapidly weak for the impact loads applied in direction other than for $\psi = 0°$. The CPU time for force and displacement methods are found to be 34 sec and 738 Sec, respectively. Thus the force method clearly provides a more efficient and computationally faster solution. The optimal results for the various values of the external impact load direction ψ are given in the Table 2.

Table 2 : Optimal values of $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and structural strength in the optimal adaptive shapes for dynamic load.

$\psi°$	Design Variables				Structural
	φ_1°	φ_2°	φ_3°	φ_4°	Strength N-Sec
-90	78.70	17.98	83.12	11.19	39.29
-80	77.69	20.03	82.40	11.64	52.29
-70	79.45	21.11	76.91	15.42	61.96
-60	77.68	23.84	72.37	19.50	75.78
-50	63.95	25.72	79.94	22.82	93.82
-40	57.32	30.08	70.55	24.94	105.83
-30	56.50	33.59	68.29	24.96	108.94
-20	51.85	37.10	60.03	29.97	108.69
-10	51.11	47.18	65.37	34.59	110.63
0	53.59	47.46	51.86	41.01	105.33
10	49.39	53.55	42.68	50.56	100.62
20	44.79	65.53	39.67	60.78	96.31
30	39.22	66.83	43.08	80.67	90.45
30	39.22	66.83	43.08	80.67	90.45
40	31.96	81.75	25.62	69.83	83.85
50	29.44	85.05	22.35	72.90	79.66
60	36.12	89.50	39.82	80.69	74.20
70	35.37	89.50	26.88	71.58	73.18
80	30.79	89.49	24.13	74.57	63.80
90	29.65	89.50	19.62	75.44	57.18

Figure 8 illustrates the changes in the structural strength for the adaptive truss structure in comparison to the fixed structure for various direction of ψ and it can also be inferred that the structural strength is improved significantly as the structure adaptively changes its geometry to accommodate the changes in loading direction. The optimal geometries of the adaptive structure are shown in the Figure 9, corresponding to the different load directions. The results are in good agreement with those in [Murotsu and Shao(1990a)], which is based on the conventional displacement method.

6 Conclusions

The integrated force method has been implemented to analyze and optimize the geometry of adaptive truss structures under static and dynamic loading. The compatibility matrix is derived directly using the displacement-deformation relation and the SVD technique. The unconditionally stable Newmark algorithm has been implemented to solve the force equations of the motion. It was found that the average-acceleration New-mark method for this type of problem may introduce numerical spurious oscillations.

The application and efficiency of the proposed method was illustrated by optimizing the topology of an adaptive truss structure using active elements in order to maintain structural strength under various loading conditions. The method has proved to be perfectly natural and efficient because the primary variables in topology optimization of adaptive truss structures are the member forces. It has been shown that the structural strength can be improved significantly by optimally changing the geometric configuration to counteract the changes in the direction of the applied load. The computational time has been reduced drastically when using the force method to analyze and optimize adaptive structures when compared to the numerical solution using the displacement method.

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